Blacktown Boys' High School

2020 Year 12

HSC Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 34)

- Attempt Questions 11 – 32
- Allow about 2 hour 45 minutes for this section

Assessor: Mrs Chirgwin

Student Name: ____

Teacher Name: ____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2020 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1. What is the domain of the function $f(x) = \sqrt{1 - x^2}$?

A. $(0, 1)$

B. $[0, 1]$

C. $(-1, 1)$

D. $[-1, 1]$

Q2. What is the angle of inclination of the line $\sqrt{3}x - y + 2\sqrt{3} = 0$ with respect to the positive x-axis?

A. $30°$

B. $60°$

C. $120°$

D. $150°$

Q3. Which of the following is equal to $\dfrac{\log_3 32}{\log_3 2}$?

A. $\log_3 30$

B. $\log_3 16$

C. 16

D. 5

Q4. John works in a cake shop, and based on sales over two weeks, he conducted a survey of the five most popular cakes. What type of data is this?

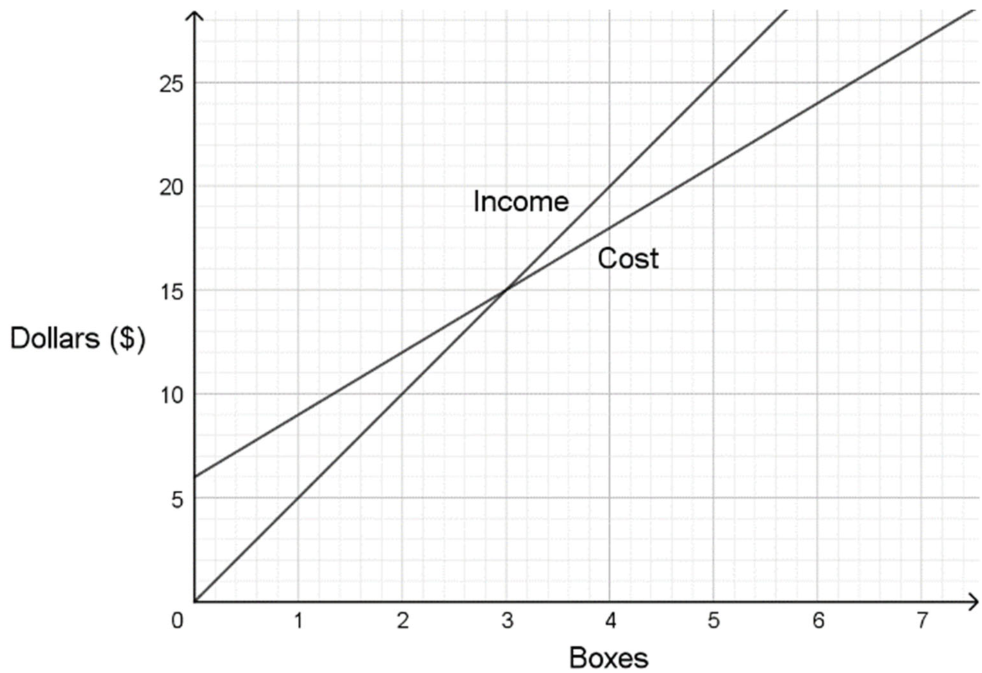
A. Categorical nominal

B. Categorical ordinal

C. Quantitative continuous

D. Quantitative discrete

Q5. The graph below shows the cost of producing boxes of chocolates and the income received from their sale.



Use the graph to determine the number of boxes that need to be sold to break even.

A. 5

B. 4

C. 3

D. 2

Q6. The time taken to travel between two regional cities is approximately normally distributed with a mean of 85 minutes and a standard deviation of 4 minutes.

The percentage of travel times that are between 81 minutes and 93 minutes is closest to

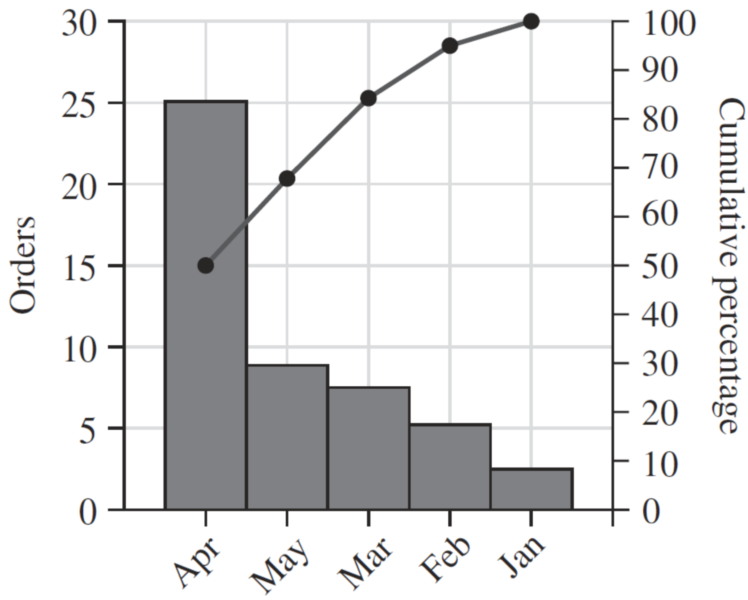
A. 68%

B. 70.5%

C. 81.5%

D. 95%

Q7. The Pareto chart below shows the order received by a business for five months.



What percentage of orders were received in May?

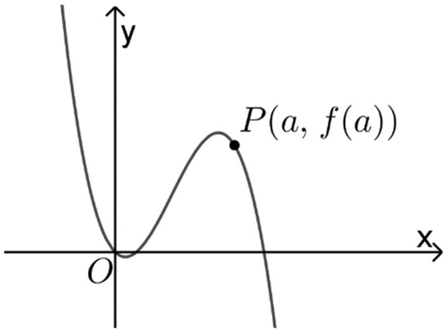
A. 69%

B. 45%

C. 30%

D. 18%

Q8. Which statement is true for the point P on the curve $y = f(x)$?



A. $f'(a) < 0, f''(a) < 0$

B. $f'(a) > 0, f''(a) > 0$

C. $f'(a) < 0, f''(a) > 0$

D. $f'(a) > 0, f''(a) < 0$

Q9. A box contains five red marbles and four blue marbles. Stella selects three marbles at random, without replacing them.

The probability that at least one of the marbles that Stella selects is red is

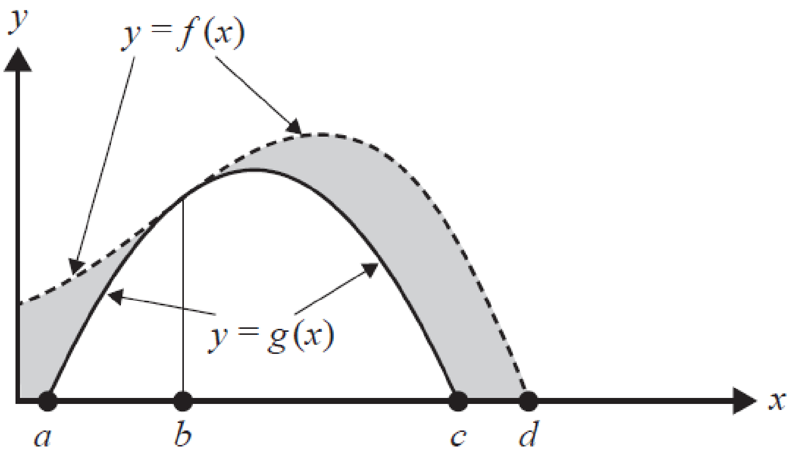
A. $\frac{5}{9}$

B. $\frac{20}{21}$

C. $\frac{5}{42}$

D. $\frac{665}{729}$

Q10. Consider the graphs of the function $f(x)$ and $g(x)$ shown below.



The area of the shaded region could be represented by

A. $\int_a^d \big(f(x) - g(x)\big)dx$

B. $\int_0^d \big(f(x) - g(x)\big)dx$

C. $\int_0^b \big(f(x) - g(x)\big)dx + \int_b^c \big(f(x) - g(x)\big)dx$

D. $\int_0^d f(x)dx - \int_a^c g(x)dx$

End of Section I

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11-32
Allow about 2 hour and 45 minutes for this section

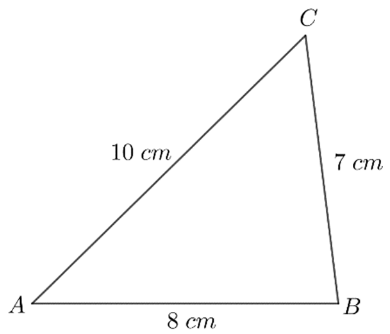
Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.

Please turn over

Question 11 (3 marks)

The diagram shows ΔABC with sides $AB = 8\ cm$, $BC = 7\ cm$ and $AC = 10\ cm$.



NOT TO SCALE

a) Show that $\cos A = \dfrac{23}{32}$. **1**

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b) By finding the exact value of $\sin A$, determine the exact value of the area of ΔABC. **2**

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Question 12 (3 Marks)

In an arithmetic series, the first term is 18 and the sum of the first 20 terms is 1310.

a) Show that the 20^{th} term is 113. **1**

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b) Find the common difference. **1**

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c) Find the sum of first 35 terms. **1**

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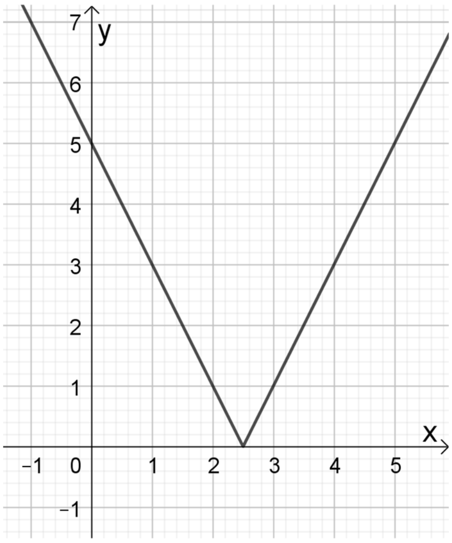
Question 13 (3 marks)

Given the graph of $f(x) = |ax + b|$



a) What are the values of a and b ? **2**

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b) Solve for $f(x) \geq 3$. **1**

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Question 14 (4 marks)

Differentiate

a) $y = \tan^3\left(\frac{x}{4}\right)$ **2**

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b) $y = \frac{e^{3x} - 5}{x + 1}$ **2**

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Question 15 (2 marks)

What is the limiting sum of the following geometric series? **2**

$$750, -300, 120, -48, \ldots$$

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Question 16 (2 marks)

The first four terms of a geometric sequence are $48, m, n$ and 750. Find the values of m and n. **2**

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Question 17 (6 marks)

Find the primitive functions of

a) $\int (5x + 3)^{19} dx$ **2**

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b) $\int \left(2x^3 + \frac{1}{3x + 1}\right) dx$ **2**

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c) $\int \left(\sin 10x - \frac{2}{e^{5x}}\right) dx$ **2**

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Question 18 (4 marks)

a) Show that $\dfrac{d}{dx}(e^{2x}\cos x) = e^{2x}(2\cos x - \sin x)$ **2**

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b) Hence find the equation of the tangent to the curve $y = e^{2x}\cos x$ at the point where $x = 0$. **2**

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Question 19 (4 marks)

The table below gives the present value of an annuity of $1 at the given interest rate for the given period.

Present Value Interest Factors (PVA)								
$1	Interest Rate Per Period							
N	1%	2%	3%	4%	5%	6%	7%	8%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9269
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121
5	4.8545	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101

a) Jesse plans to invest $7500 per year for 8 years in an annuity. His investment will earn interest at the rate of 6% per annum. Calculate the present value of his annuity. **2**

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b) Shaon takes out a loan of $12000 to buy a car. This loan is to be repaid over 5 years at an interest rate of 8% per year. Use the PVA table to find his yearly repayments. **2**

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Question 20 (2 marks)

The curve $y = f(x)$ passes through the point $(-1, 2)$ and $f'(x) = 4x^3 - 3$. **2**

Find $f(x)$.

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Question 21 (3 marks)

Show that $\int_0^2 \sqrt{4x+1}\,dx = \frac{13}{3}$ **3**

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Question 22 (3 marks)

Solve $4\cos^2 x - 3 = 0$ for $-\pi \leq x \leq \pi$. **3**

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Question 23 (3 marks)

A discrete random variable X has the probability distribution table shown. **3**

$X = x$	20	21	22	23
$P(x)$	0.24	0.2	m	0.4

By finding the value of m, calculate the expected value and the variance of X.

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Question 24 (3 marks)

Given that $y = \log_e(2^x - x)$

a) Complete the table below by finding the missing value of y to 3 decimal places. **1**

x	1	1.5	2	2.5	3
y	0	0.284	0.693		1.609

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b) Use the trapezoidal rule with all the values of y from the table above to find and approximation for the value of **2**

$$\int_1^3 \log_e(2^x - x)\,dx$$

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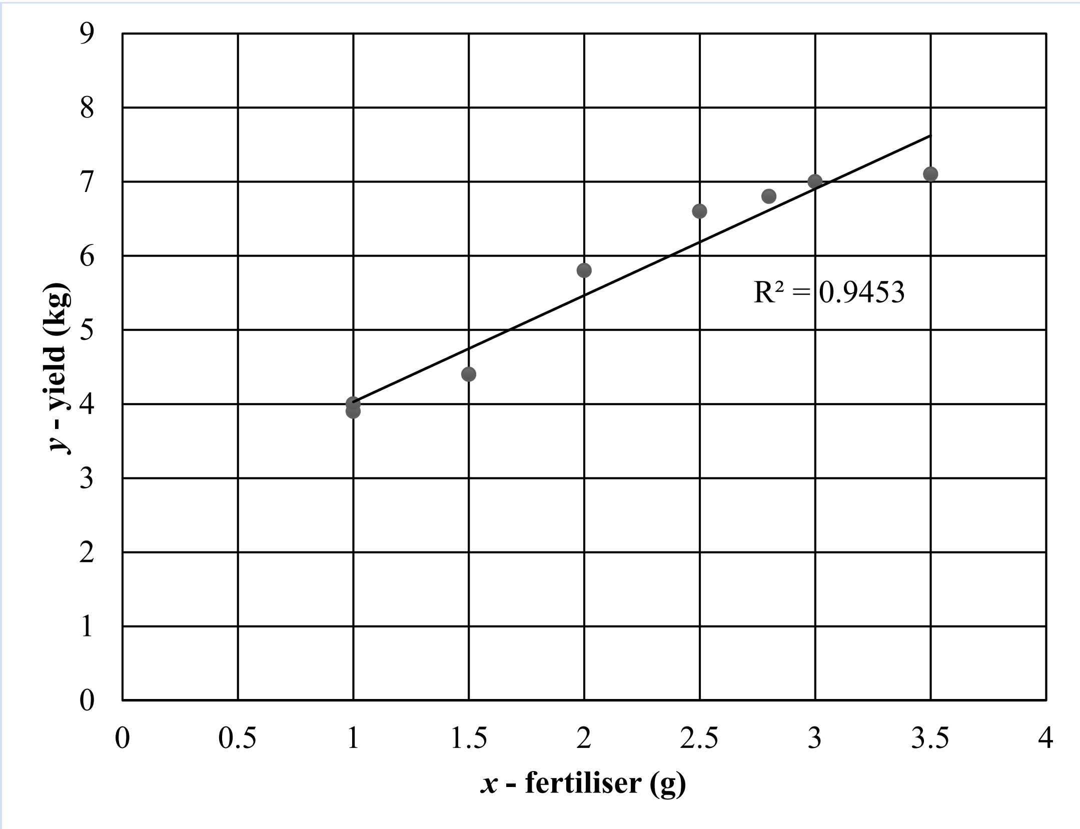
Question 25 (5 marks)

A biologist assumes that there is a linear relationship between the amount of fertiliser supplied to tomato plants and the subsequent yield of tomatoes obtained.

Eight tomato plants, of the same variety, were selected at random and treated, weekly, with a solution in which x grams of fertiliser was dissolved in a fixed quantity of water. The yield, y kilograms, of tomatoes was recorded.

Plant	A	B	C	D	E	F	G	H
x	1.0	1.0	1.5	2.0	2.5	2.8	3.0	3.5
y	3.9	4.0	4.4	5.8	6.6	6.8	7.0	7.1



a) Use the scatterplot to describe the association between 'yield' and 'fertiliser' in terms of strength and direction. **1**

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Question 25 continues on next page

Question 25 (continued)

b) Determine the equation of the least-squares regression line for this data. Round your values to two significant figures. **2**

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c) A plant with 2.2 grams of fertiliser was not recorded by accident. **1**

Calculate the predicted yield for this plant using your answer in part b).

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d) Explain why you should not extrapolate from this data to find the yield for high rates of fertiliser usage. **1**

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Question 26 (7 marks)

Consider the curve given by $y = 3x - x^3 - 1$, for $-3 \leq x \leq 2$.

a) Find the stationary points and determine their nature. **3**

Question 26 continues on next page

Question 26 (continued)

b) Find the point of inflection. **1**

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c) Sketch the curve for $-3 \leq x \leq 2$, showing all key features. **2**

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d) What is the maximum value of this function for $-3 \leq x \leq 2$. **1**

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Question 27 (5 marks)

The chocolate consumption per person per day of a population of people was found to be normally distributed with a mean of 68.95 grams and a standard deviation of 18.45.

a) Above what chocolate consumption rate does 2.5% of this population lie? **1**

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b) Rahul consumed 50.5 grams of chocolate in one day. What percentage of this population have a chocolate consumption rate more than Rahul's? **2**

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c) If Ben takes a sample of people from this population and finds that 6 of them consumed less than 13.6 grams of chocolate per person per day. If Ben's sample has the same distribution as this population, what is Ben's sample size? **2**

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Question 28 (5 marks)

The length of time, in minutes, that a customer queues to buy toilet paper is a random variable, X, with probability density function

$$f(x) = \begin{cases} k(64 - x^2) & \text{for } 0 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

a) Show that the value of k is $\dfrac{3}{1024}$. **2**

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Question 28 continues on next page

Question 28 (continued)

b) Find the cumulative distribution function. **2**

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c) Find the probability that a customer will queue for longer than 5 minutes, round your answer to 4 significant figures. **1**

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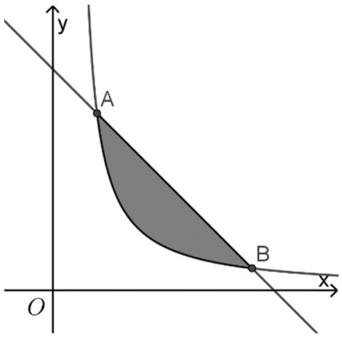
Question 29 (3 marks)

The diagram below shows the graphs of $y = 5 - x$ and $y = \dfrac{4}{2x-1}$



a) Show that the x-coordinate values of A and B are 1 and 4.5 respectively. **1**

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b) Find the exact area of the shaded region. **3**

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Question 30 (5 marks)

A gardener plants a bed of roses. The bed is planted so that the first row has 24 rose plants. The second row has 29 rose plants. Each succeeding row has 5 more rose plants than the previous row.

a) Calculate the number of rose plants in the 8^{th} row. **1**

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b) Which row would be the first to contain more than 150 rose plants? **2**

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Question 30 continues on next page

Question 30 (continued)

c) The gardener has planted 2895 rose plants altogether. Assuming that the above pattern has been continued, how many rows were planted? **2**

Question 31 (8 marks)

A particle moves in a straight line. At time t seconds its displacement is x metres from the origin O on the line. The velocity of this particle is given by

$$v = 2 - 4\cos 2t, \qquad 0 \leq t \leq 2\pi$$

a) Find the initial velocity of this particle. **1**

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b) Find all the times when the particle is at rest, where $0 \leq t \leq 2\pi$. **2**

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Question 31 continued on next page

Question 31 (continued)

c) Sketch the graph of v as a function in terms of t, showing all key features. **2**

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d) Find the acceleration of the particle when $t = \dfrac{\pi}{2}$. **1**

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Question 31 continued on next page

Question 31 (continued)

e) Find the exact displacement of this particle when $t = \pi$ given that its initial displacement is 3 metres to the right of the origin. **2**

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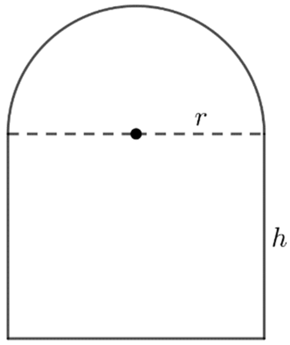
Question 32 (6 marks)

A window in the chapel has been damaged by a storm and needs to be replaced. It is in the shape of a rectangle surmounted by a semi-circle, as shown.



NOT TO SCALE

Let the radius of the semi-circle be r metres and the height of the rectangle be h metres.

a) Given that the perimeter of the window is to be 16π metres, show that **1**

$$h = 8\pi - r - \frac{1}{2}\pi r$$

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b) Hence, show that the area A of the window is given by the formula **1**

$$A = 16\pi r - 2r^2 - \frac{1}{2}\pi r^2$$

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Question 32 continues on next page

Question 32 (continued)

c) Find the exact radius of the semi-circle for which the area of the window is to be a maximum. **3**

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d) Find the maximum area of this window correct to 1 decimal place. **1**

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End of Paper

2020 Year 12 Mathematics Advanced AT4 Solutions

Section 1		
Q1	**D** The domain of semi-circle $f(x) = \sqrt{1 - x^2}$ with radius 1 is $-1 \le x \le 1$, in interval notation is $[-1, 1]$	1 Mark
Q2	**B** $\sqrt{3}x - y + 2\sqrt{3} = 0$ $y = \sqrt{3}x + 2\sqrt{3}$ $\tan\theta = \sqrt{3}$ $\theta = 60°$	1 Mark
Q3	**D** $\frac{\log_3 32}{\log_3 2}$ $= \frac{\log_3 2^5}{\log_3 2}$ $= \frac{5\log_3 2}{\log_3 2}$ $= 5$	1 Mark
Q4	**A** Type of cakes is categorical nominal	1 Mark
Q5	**C** Break even point = point of intersection Need to sell 3 boxes to break even.	1 Mark
Q6	**C** 81 is 1 sd below a mean of 85, and 93 is 2 sd above a mean of 85. $68\% + \frac{95\% - 68\%}{2}$ $= 81.5\%$	1 Mark
Q7	**D** $68\% - 50\% = 18\%$ or $\frac{9}{25 + 9 + 8 + 5 + 3} = 18\%$	1 Mark
Q8	**A** P is decreasing on the curve, so $f'(a) < 0$ P is also along there curve where it is concaving down, so $f''(a) < 0$	1 Mark
Q9	**B** Probability of at least one red marble is the complement to probability of no red marbles $P(X \ge 1) = 1 - P(X = 0)$ $P(X \ge 1) = 1 - \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}$ $P(X \ge 1) = \frac{20}{21}$	1 Mark
Q10	**D** $\int_0^d f(x)dx - \int_a^c g(x)dx$	1 Mark

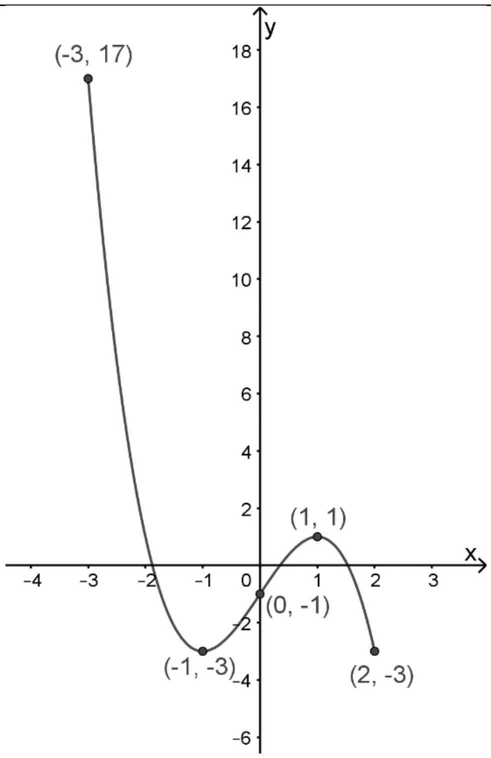
Section 2		
Q11a	$\cos A = \frac{8^2 + 10^2 - 7^2}{2 \times 8 \times 10}$ $\cos A = \frac{23}{32}$	1 Mark Correct solution
Q11b	$\sin A = \frac{\sqrt{32^2 - 23^2}}{32}$ $\sin A = \frac{\sqrt{495}}{32}$ $\sin A = \frac{3\sqrt{55}}{32}$ $Area = \frac{1}{2} \times 8 \times 10 \times \sin A$ $Area = \frac{1}{2} \times 8 \times 10 \times \frac{3\sqrt{55}}{32}$ $Area = \frac{15\sqrt{55}}{4}\ \ units^2$	2 Marks Correct solution 1 Mark Finds the exact value of $\sin A$
Q12a	$S_n = \frac{n}{2}(a + l)$ $S_{20} = \frac{20}{2}(18 + l)$ $1310 = 10(18 + l)$ $131 = 18 + l$ $l = 131 - 18$ $l = 113$	1 Mark Correct solution
Q12b	$T_n = a + (n - 1)d$ $T_{20} = 18 + 19d$ $113 = 18 + 19d$ $95 = 19d$ $d = 5$	1 Mark Correct solution
Q12c	$S_n = \frac{n}{2}(2a + (n - 1)d)$ $S_{35} = \frac{35}{2}(2 \times 18 + (35 - 1) \times 5)$ $S_{35} = 3605$	1 Mark Correct solution
Q13a	x intercept at 2.5 $2.5a - 5 = 0$ $2.5a = 5$ $a = 2$ y intercept at 5 $\|0 + b\| = 5$ $b = \pm 5$ Check a point $(2, 1)$ $1 \neq \|2 \times 2 + 5\|$ $1 = \|2 \times 2 - 5\|$ $\therefore b = -5$ $\therefore a = 2, b = -5$	2 Marks Correct solution 1 Mark Correct value of a or b
Q13b	$\|2x - 5\| \geq 3$ $x \leq 1, x \geq 4$	1 Mark Correct solution

Q14a	$y = \tan^3\left(\frac{x}{4}\right)$ $\frac{dy}{dx} = 3 \times \frac{1}{4}\sec^2\left(\frac{x}{4}\right) \times \tan^2\left(\frac{x}{4}\right)$ $\frac{dy}{dx} = \frac{3}{4}\sec^2\left(\frac{x}{4}\right)\tan^2\left(\frac{x}{4}\right)$	2 Marks Correct solution 1 Mark Correct differentiation of $\tan\frac{x}{4}$
Q14b	$y = \frac{e^{3x} - 5}{x + 1}$ $\frac{dy}{dx} = \frac{(x+1) \times 3e^{3x} - (e^{3x} - 5) \times 1}{(x+1)^2}$ $\frac{dy}{dx} = \frac{3xe^{3x} + 3e^{3x} - e^{3x} + 5}{(x+1)^2}$ $\frac{dy}{dx} = \frac{3xe^{3x} + 2e^{3x} + 5}{(x+1)^2}$	2 Marks Correct solution 1 Mark Correct differentiation of $e^{3x} - 5$
Q15	$750, -300, 120, -48, \ldots$ $a = 750$ $r = -\frac{300}{750}$ $r = -\frac{2}{5}$ $S = \frac{a}{1 - r}$ $S = \frac{750}{1 - \left(-\frac{2}{5}\right)}$ $S = \frac{3750}{7}$	2 Marks Correct solution 1 Mark Finds the correct value of r
Q16	$a = 48$ (1) $ar = m$ (2) $ar^2 = n$ (3) $ar^3 = 750$ (4) Sub (1) into (4) $48r^3 = 750$ $r^3 = \frac{125}{8}$ $r = \sqrt[3]{\frac{125}{8}}$ $r = \frac{5}{2}$ $m = 48 \times \frac{5}{2}$ $m = 120$ $n = 48 \times \left(\frac{5}{2}\right)^2$ $n = 300$	2 Marks Correct solution 1 Mark Finds the correct value of r

Q17a	$\int (5x+3)^{19}dx$ $= \frac{(5x+3)^{20}}{20 \times 5} + C$ $= \frac{(5x+3)^{20}}{100} + C$	2 Marks Correct solution 1 Mark Correct integration without C
Q17b	$\int \left(2x^3 + \frac{1}{3x+1}\right) dx$ $= \frac{2x^4}{4} + \frac{1}{3}\ln\|3x+1\| + C$ $= \frac{x^4}{2} + \frac{1}{3}\ln\|3x+1\| + C$	2 Marks Correct solution 1 Mark Correct integration of $\frac{1}{3x+1}$
Q17c	$\int \left(\sin 10x - \frac{2}{e^{5x}}\right) dx$ $= \int (\sin 10x - 2e^{-5x})dx$ $= -\frac{1}{10}\cos 10x - \left(-\frac{2}{5}e^{-5x}\right) + C$ $= \frac{2}{5e^{5x}} - \frac{1}{10}\cos 10x + C$	2 Marks Correct solution 1 Mark Correct integration of $\sin 10x$ or $\frac{2}{e^{5x}}$
Q18a	$y = e^{2x}\cos x$ $\frac{dy}{dx} = e^{2x} \times -\sin x + 2e^{2x}\cos x$ $\frac{dy}{dx} = -e^{2x}\sin x + 2e^{2x}\cos x$ $\frac{dy}{dx} = e^{2x}(2\cos x - \sin x)$	2 Marks Correct solution 1 Mark Makes significant progress
Q18b	At $x = 0$ $m_T = e^{2\times0} \times (2\cos 0 - \sin 0)$ $m_T = 2$ $y = e^0 \times \cos 0$ $y = 1$ Equation of tangent $y - 1 = 2(x - 0)$ $y - 1 = 2x$ $2x - y + 1 = 0$	2 Marks Correct solution 1 Mark Finds the gradient of the tangent
Q19a	8 years at 6% gives the factor of 6.2098 $PVA = 7500 \times 6.2098$ $PVA = \$46573.50$	2 Marks Correct solution 1 Mark Correct PVA factor value
Q19b	Let M be the yearly repayments. 5 years at 8% gives the factor of 3.9927 $3.9927M = 12000$ $M = \frac{12000}{3.9927}$ $M = \$3005.48501\ldots$ $M = \$3005.49$	2 Marks Correct solution 1 Mark Establishes $3.9927M = 12000$

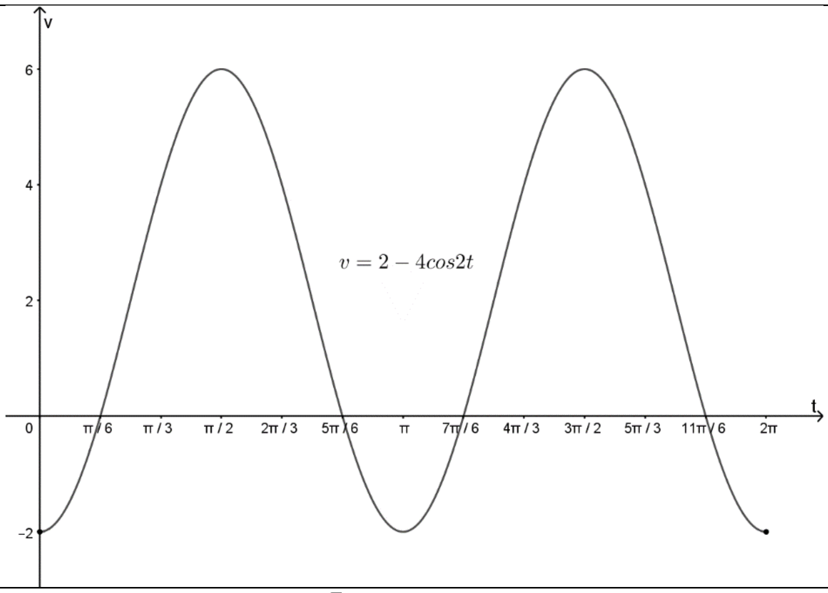
Q20	$f'(x) = 4x^3 - 3$ $f(x) = \int (4x^3 - 3)dx$ $f(x) = \frac{4x^4}{4} - 3x + C$ $f(x) = x^4 - 3x + C$ At $(-1, 2)$ $2 = (-1)^4 - 3(-1) + C$ $2 = 1 + 3 + C$ $C = -2$ $f(x) = x^4 - 3x - 2$	2 Marks Correct solution 1 Mark Correct primitive function
Q21	$\int_0^2 \sqrt{4x+1}dx$ $= \int_0^2 (4x+1)^{\frac{1}{2}}dx$ $= \left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2} \times 4}\right]_0^2$ $= \left[\frac{(4x+1)^{\frac{3}{2}}}{6}\right]_0^2$ $= \frac{(4 \times 2 + 1)^{\frac{3}{2}}}{6} - \frac{(4 \times 0 + 1)^{\frac{3}{2}}}{6}$ $= \frac{27}{6} - \frac{1}{6}$ $= \frac{13}{3}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Correct integration
Q22	$4\cos^2 x - 3 = 0$ $\cos^2 x = \frac{3}{4}$ $\cos x = \pm\frac{\sqrt{3}}{2}$ $x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Correctly obtains $\cos x = \pm\frac{\sqrt{3}}{2}$
Q23	$0.24 + 0.2 + m + 0.4 = 1$ $m = 0.16$ $E(X) = 20 \times 0.24 + 21 \times 0.2 + 22 \times 0.16 + 23 \times 0.4$ $E(X) = 21.72$ $Var(X) = E(X^2) - [E(X)]^2$ $Var(X) = 20^2 \times 0.24 + 21^2 \times 0.2 + 22^2 \times 0.16 + 23^2 \times 0.4 - 21.72^2$ $Var(X) = 1.4816$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Finds the correct value of m
Q24a	$y = \log_e(2^{2.5} - 2.5)$ $y = 1.14957 \ldots$ $y = 1.150$	1 Mark Correct solution

Q24b	$h = 0.5$ $A = \int_1^3 \log_e(2^x - x)\,dx$ $A \approx \frac{0.5}{2}\big(0 + 1.609 + 2 \times (0.284 + 0.693 + 1.150)\big)$ $A \approx 1.46575$	2 Marks Correct solution 1 Mark Makes significant progress
Q25a	Linear, strong, positive	1 Mark Correct solution
Q25b	Calculator – stat mode $y = A + Bx$ $A = 2.5920 \ldots$ $B = 1.4371 \ldots$ $y = 1.4x + 2.6$ Or $m = \frac{5.5 - 4}{2 - 1} = 1.5$ $y - 4 = 1.5(x - 1)$ $y = 1.5x + 2.5$	2 Marks Correct solution 1 Mark Makes significant progress
Q25c	$y = 1.4 \times 2.2 + 2.6$ Or $y = 1.5 \times 2.2 + 2.5$ $y = 5.68\ kg$ $y = 5.8\ kg$	1 Mark Correct solution
Q25d	There could be a point where fertiliser will not produce extra yield. Reasons could include: high volume of fertiliser may cause damage to the plant; other factors may limit yield, eg sunlight/water; plants may not be able to absorb fertiliser at that rate.	1 Marks Correct solution
Q26a	$y = 3x - x^3 - 1$ $y' = 3 - 3x^2$ $y'' = -6x$ For stationary points, $y' = 0$ $3 - 3x^2 = 0$ $x^2 = 1$ $x = \pm 1$ $x = 1$ $y = 3 \times 1 - 1^3 - 1$ $y = 1$ $y'' = -6 \times 1$ $y'' = -6$ $y'' < 0$ $\therefore$ $(1, 1)$ is a maximum turning point. $x = -1$ $y = 3 \times (-1) - (-1)^3 - 1$ $y = -3$ $y'' = -6 \times -1$ $y'' = 6$ $y'' > 0$ $\therefore$ $(-1, -3)$ is a minimum turning point.	3 Marks Correct solution 2 Marks Finds the coordinates of all the stationary points 1 Mark Finds the correct values of x for the stationary points

Q26b	$y'' = -6x$ $-6x = 0$ $x = 0$ $y = 3x - x^3 - 1$ $y = -1$ <table><tr><td>x</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>y''</td><td>6</td><td>0</td><td>-6</td></tr><tr><td></td><td>$\cup$</td><td>*</td><td>$\cap$</td></tr></table> Since there is a change in concavity, therefore $(0, -1)$ is a point of inflection.	1 Mark Correct solution
Q26c		2 Marks Correct solution 1 Mark Correct sketch showing some key features
Q26d	The maximum value of y is 17 in the domain of $-3 \leq x \leq 2$.	1 Mark Correct solution
Q27a	2.5% is a 2 z-score above the mean $68.95 + 2 \times 18.45 = 105.85$ grams 2.5% of this population consumed more than 105.85 grams of chocolate per person per day.	1 Mark Correct solution
Q27b	$z = \dfrac{50.5 - 68.95}{18.45}$ $z = -1$ $\dfrac{1}{2}(100\% - 68\%)$ $= 16\%$ $100\% - 16\% = 84\%$ $\therefore$ 84% of people consumed more than 50.5 grams of chocolate per person per day.	2 Marks Correct solution 1 Mark Correct z score calculated

Q27c	$z = \frac{13.6 - 68.95}{18.45}$ $z = -3$ $\frac{1}{2}(100\% - 99.7\%)$ $= 0.15\%$ 0.15% represents 6 people $\frac{6}{0.15\%}$ $= 4000$ ∴ Sample size is 4000 people.	2 Marks Correct solution 1 Mark Makes significant progress
Q28a	$\int_0^8 k(64 - x^2) = 1$ $k\left[64x - \frac{x^3}{3}\right]_0^8 = 1$ $k\left[64 \times 8 - \frac{8^3}{3} - 0\right] = 1$ $k \times \frac{1024}{3} = 1$ $k = \frac{3}{1024}$	2 Marks Correct solution 1 Mark Correct primitive function
Q28b	$f(x) = \begin{cases} \frac{3}{1024}(64 - x^2) & \text{for } 0 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$ $F(x) = \int_0^x \frac{3}{1024}(64 - x^2)dx$ $F(x) = \frac{3}{1024}\left[64x - \frac{x^3}{3}\right]_0^x$ $F(x) = \frac{3}{1024}\left[64x - \frac{x^3}{3}\right]_0^x$ $F(x) = \frac{3}{1024}\left(64x - \frac{x^3}{3}\right)$ $F(x) = \frac{3x}{16} - \frac{x^3}{1024}$ $\therefore F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x}{16} - \frac{x^3}{1024} & 0 \le x \le 8 \\ 1 & x > 8 \end{cases}$	2 Marks Correct solution 1 Mark Correct primitive function
Q28c	$P(X > 5) = 1 - P(X \le 5)$ $P(X > 5) = 1 - F(5)$ $P(X > 5) = 1 - \left(\frac{3}{16} \times 5 - \frac{5^3}{1024}\right)$ $P(X > 5) = \frac{189}{1024}$ $P(X > 5) = 0.184570 \ldots$ $P(X > 5) = 0.1846$ (4 sig figs)	1 Mark Correct solution

Q29a	$y_1 = 5 - x$ $y_2 = \frac{4}{2x-1}$ Sub $x = 1$ $y_1 = 5 - 1 = 4$ $y_2 = \frac{4}{2 \times 1 - 1} = 4$ $y_1 = y_2$ Sub $x = 4.5$ $y_1 = 5 - 4.5 = 0.5$ $y_2 = \frac{4}{2 \times 4.5 - 1} = 0.5$ $y_1 = y_2$ Or solve simultaneously $5 - x = \frac{4}{2x-1}$ $(5-x)(2x-1) = 4$ $10x - 5 - 2x^2 + x = 4$ $-2x^2 + 11x - 5 - 4 = 0$ $2x^2 - 11x + 9 = 0$ $(2x-9)(x-1) = 0$ $x = 1, x = 4.5$	1 Mark Correct solution
Q29b	$A = \int_1^{4.5} \left(5 - x - \frac{4}{2x-1}\right) dx$ $A = \left[5x - \frac{x^2}{2} - 2\ln\lvert 2x-1\rvert\right]_1^{4.5}$ $A = \left(5 \times 4.5 - \frac{4.5^2}{2} - 2\ln\lvert 2 \times 4.5 - 1\rvert\right) - \left(5 \times 1 - \frac{1^2}{2} - 2\ln\lvert 2 \times 1 - 1\rvert\right)$ $A = \left(\frac{99}{8} - 2\ln 8\right) - \left(\frac{9}{2} - 2\ln 1\right)$ $A = \left(\frac{63}{8} - 2\ln 8\right)\ units^2$	3 Marks Correct solution 2 Marks Correct primitive function 1 Mark Expresses area correctly using the definite integral
Q30a	This is an AP: $a = 24, d = 5$ $T_8 = 24 + (8-1) \times 5$ $T_8 = 59$	1 Mark Correct solution
Q30b	$24 + (n-1) \times 5 > 150$ $24 + 5n - 5 > 150$ $5n > 131$ $n > \frac{131}{5}$ $n > 26\frac{1}{5}$ $n = 27$ ∴ The 27th row is the first row to contain more than 150 rose plants.	2 Marks Correct solution 1 Mark Makes significant progress

Q30c	$2895 = \frac{n}{2}(2 \times 24 + (n-1) \times 5)$ $5790 = n(48 + 5n - 5)$ $5790 = 43n + 5n^2$ $5n^2 + 43n - 5790 = 0$ $(n - 30)(5n + 193) = 0$ $n = 30, n = -\frac{193}{5}$ $\therefore$ 30 rows were planted.	2 Marks Correct solution 1 Mark Makes significant progress
Q31a	Initial velocity $t = 0$ $v = 2 - 4\cos 0$ $v = 2 - 4$ $v = -2$ Initial velocity of the particle is $-2\ m/s$.	1 Mark Correct solution
Q31b	At rest $v = 0$ $2 - 4\cos 2t = 0$ $4\cos 2t = 2$ $\cos 2t = \frac{1}{2}$ $0 \le t \le 2\pi$ $0 \le 2t \le 4\pi$ $2t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ $t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	2 Marks Correct solution 1 Mark Makes significant progress
Q31c		2 Marks Correct solution 1 Mark Correct graph with some key features shown
Q31d	From the graph, when $t = \frac{\pi}{2}$, maximum turning point for the velocity function, indicates acceleration is $0\ m/s^2$. OR $\ddot{x} = \frac{dv}{dt}$ $\ddot{x} = 8\sin 2t$ $t = \frac{\pi}{2}$ $\ddot{x} = 8\sin 2t$ $\ddot{x} = 0\ m/s^2$	1 Mark Correct solution

Q31e	$\frac{dx}{dt} = 2 - 4\cos 2t$ $x = \int (2 - 4\cos 2t)dt$ $x = 2t - 2\sin 2t + C$ $t = 0, x = 3$ $3 = 0 - 2\sin 0 + C$ $C = 3$ $x = 2t - 2\sin 2t + 3$ When $t = \pi$ $x = 2\pi - 2\sin 2\pi + 3$ $x = 2\pi + 3$ The displacement of this particle is $2\pi + 3$ metres to the right of the origin when $t = \pi$.	2 Marks Correct solution 1 Mark Correct primitive function
Q32a	Perimeter is $2h + 2r + \pi r$ $16\pi = 2h + 2r + \pi r$ $2h = 16\pi - 2r - \pi r$ $h = 8\pi - r - \frac{1}{2}\pi r$	1 Mark Correct solution
Q32b	$A = 2rh + \frac{1}{2}\pi r^2$ $A = 2r\left(8\pi - r - \frac{\pi r}{2}\right) + \frac{\pi r^2}{2}$ $A = 16\pi r - 2r^2 - \pi r^2 + \frac{\pi r^2}{2}$ $A = 16\pi r - 2r^2 - \frac{\pi r^2}{2}$	1 Mark Correct solution
Q32c	$\frac{dA}{dr} = 16\pi - 4r - \pi r$ $\frac{dA}{dr} = 0$ $16\pi - 4r - \pi r = 0$ $4r + \pi r = 16\pi$ $r(4 + \pi) = 16\pi$ $r = \frac{16\pi}{4 + \pi}$ $\frac{d^2A}{dr^2} = -4 - \pi$ $\frac{d^2A}{dr^2} < 0$ $\therefore r = \frac{16\pi}{4+\pi}$ produces a maximum area.	3 Marks Correct solution 2 Marks Finds the exact value of r 1 Mark Correct differentiation
Q32d	$A = 16\pi \times \left(\frac{16\pi}{4 + \pi}\right) - 2 \times \left(\frac{16\pi}{4 + \pi}\right)^2 - \frac{\pi \times \left(\frac{16\pi}{4 + \pi}\right)^2}{2}$ $A = 176.8946 \ldots$ $A = 176.9\ m^2$ $\therefore$ maximum area of this window is $176.9\ m^2$	1 Mark Correct solution