



**BAULKHAM HILLS
HIGH
SCHOOL**

2018

**YEAR 11
HALF YEARLY
ASSESSMENTS**

Mathematics

**General
Instructions**

- Reading time – 5 minutes
- Working time – 2 hours 30 minutes
- Write using black or blue pen
Black pen is preferred
- Board-approved calculators may be used
- In Questions 11 – 17, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

**Total marks:
82**

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 72 marks (pages 6 – 12)

- Attempt Questions 11 – 16
- Allow about 2 hours 15 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 Which one of the following expressions represents the factored form of $8x^3 + 27$?

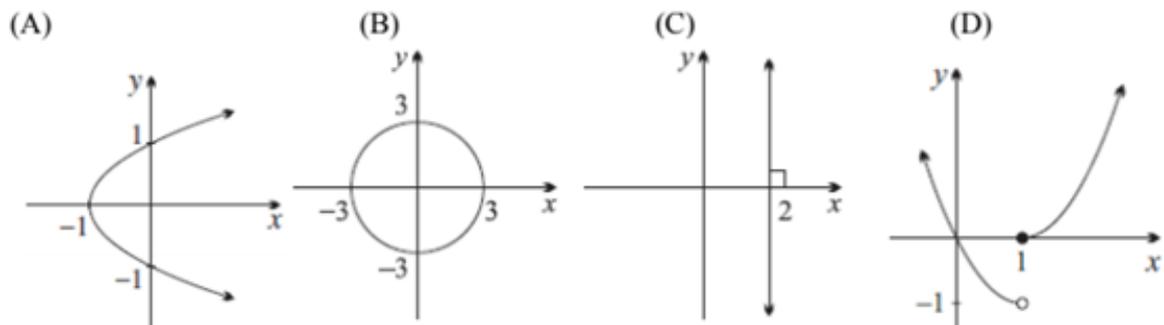
(A) $(2x + 3)(4x^2 + 6x + 9)$

(B) $(2x + 3)(4x^2 - 6x + 9)$

(C) $(2x - 3)(4x^2 - 6x - 9)$

(D) $(2x - 3)(4x^2 + 6x - 9)$

2 Which of the following is a function?



3 $\frac{1 + \sqrt{3}}{2 - \sqrt{3}} =$

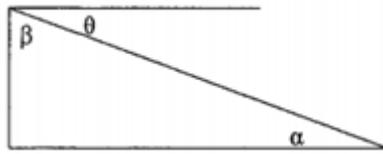
(A) $5 + 3\sqrt{3}$

(B) $\frac{5 + 3\sqrt{3}}{7}$

(C) $\sqrt{3} - 1$

(D) $\frac{\sqrt{3} - 1}{7}$

4



In the diagram above;

- (A) The angle of elevation is α and the angle of depression is β
- (B) The angle of elevation is β and the angle of depression is θ
- (C) The angle of elevation is θ and the angle of depression is β
- (D) The angle of elevation is α and the angle of depression is θ

5 Given $\cos\theta = -\frac{3}{5}$ and $\sin\theta < 0$, the ratio for $\tan\theta =$

- (A) $-\frac{4}{5}$
- (B) $-\frac{4}{3}$
- (C) $\frac{4}{3}$
- (D) $\frac{4}{5}$

- 6 In simplified form, the algebraic expression $\frac{x+5}{(x-3)(x+1)} - \frac{x-1}{x^2-x-2}$ can be written as

(A) $\frac{7x-13}{(x+1)(x-2)(x-3)}$

(B) $\frac{-x-13}{(x+1)(x-2)(x-3)}$

(C) $\frac{7x-7}{(x+1)(x-2)(x-3)}$

(D) $\frac{-x-7}{(x+1)(x-2)(x-3)}$

7 $\frac{\sin(360^\circ - A)}{\sin(90^\circ - A)} =$

- (A) -1
(B) 1
(C) - tanA
(D) tanA

- 8 Two functions are defined as $f(x) = 3x^2 - 4$ and $g(y) = y^2 - 2y$. How many solutions are there to the equation $f(a) = g(2a)$?

- (A) 0
(B) 1
(C) 2
(D) 3

- 9 The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k ?

- (A) 5
(B) 6
(C) 25
(D) 36
- 10 Let r and s be integers, then $\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$ is an integer if
- (A) $r + s \leq 0$
(B) $s \leq 0$
(C) $r \leq 0$
(D) $r \geq s$

END OF SECTION I

Section II

72 marks

Attempt Questions 11 – 16

Allow about 2 hours 15 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

In Questions 11 to 16, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (12 marks) Use a *separate* answer sheet

(a) Write $\sqrt[4]{x^5}$ in index form. 1

(b) Evaluate $\sqrt{\frac{4.81 \times 10^5}{7.36 \times 10^9}}$ correct to two significant figures. 2

(c) If $f(x) = 7 - 2x^2$, find the value of $f(-1)$. 1

(d) Simplify $\sqrt{75} - 2\sqrt{27}$. 2

(e) Expand and simplify $(3x - 4)(x - 2)(x + 2)$. 2

(f) Factorise

(i) $2x^2 + 3x - 2$ 1

(ii) $x^3 + 5x^2 + x + 5$ 1

(iii) $4a^2(x^3 + 18ab^2) - (32a^5 + 9b^2x^3)$ 2

Question 12 (12 marks) Use a *separate* answer sheet

(a) Express as a single fraction in simplest terms

(i) $\frac{125a^3 - 8}{a^2 - 7a + 10} \times \frac{a - 5}{25a^2 - 4}$ 2

(ii) $\frac{2}{x^2 - 1} - \frac{1}{x^2 - x} + \frac{x - 1}{x^2 + x}$ 3

(b) Solve

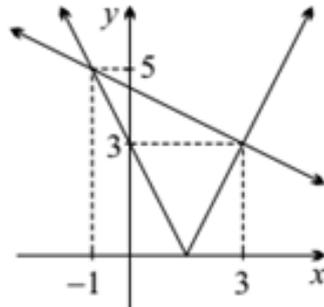
(i) $\frac{3x - 2}{4} - \frac{2x - 1}{8} = 5$ 2

(ii) $|2x + 6| = 3x - 1$ 3

(iii) Solve $2x^2 - 6x \leq 0$ 2

Question 13 (12 marks) Use a *separate* answer sheet

- (a) The graph below shows the absolute value function $y = |2x - 3|$ and the straight line $y = \frac{9-x}{2}$. 1



Use the graph to solve the inequation $|2x - 3| \geq \frac{9-x}{2}$.

- (b) Rationalise the denominator of $\frac{10}{\sqrt{5}-\sqrt{3}}$. 2

- (c) Express $0.01\dot{2}\dot{3}$ as a fraction in its simplest form, without the aid of a calculator. 2

- (d) Make v the subject of the formula $F = \frac{mv^2}{gr}$ 2

- (e) Use the method of “completing the square” to solve the equation 2

$$x^2 - 8x + 9 = 0$$

- (f) For the equation $(x + 2y)(2x - y) + (x - y)(3x + 4y) = 22$

- (i) Verify that $x = 2$ and $y = 1$ is a possible solution 1

- (ii) Find any other value(s) of x which makes the equation true when $y = 1$ 2

Question 14 (12 marks) Use a *separate* answer sheet

(a) Consider the function $y = f(x)$

(i) State the condition for the function to be even. 1

(ii) Give an example of an even function. 1

(b) The function $f(x)$ is defined as 2

$$f(x) = \begin{cases} x^2 + 1 & : x > 3 \\ 3x & : -2 \leq x \leq 3 \\ 2 & : x < -2 \end{cases}$$

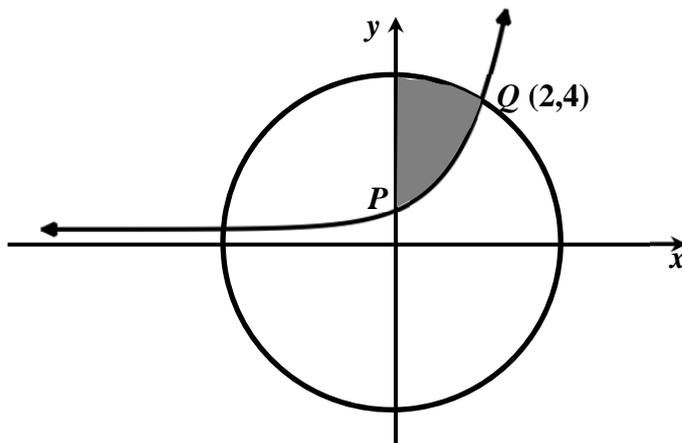
Find $f(2) + f(5) - f(-2)$.

(c) Consider the function $y = \sqrt{x^2 - 16}$

(i) Find the domain of the function. 1

(ii) Find the range of the function. 1

(d) A circle, centred at the origin and an exponential of the form $y = a^x$ are shown below.



P and Q lie on the curves as shown, Q has coordinates $(2,4)$

(i) Write down the coordinates of P . 1

(ii) Find the equation of the circle. 2

(iii) Find the equation of the exponential function. 1

(iv) Give the three inequations which combine to define the shaded region. 2

Question 15 (12 marks) Use a *separate* answer sheet

(a) It is known that $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

Using this value, find the following;

(Note: your working *MUST* show how you used $\sin 15^\circ$)

(i) $\cos 75^\circ$. 1

(ii) $\operatorname{cosec} 15^\circ$. 1

(iii) $\sin 195^\circ$. 1

(b) Solve the simultaneous equations 2

$$\begin{aligned} 2x + y &= 4 \\ 5x + 2y &= 9 \end{aligned}$$

(c) Prove the relationship $\sin^4 x - \cos^4 x \equiv 1 - 2\cos^2 x$. 2

(d) A student is asked to solve $\tan x = \sin x$, giving all solutions in the range $0^\circ \leq x \leq 360^\circ$.
Below is the student's working;

$$\begin{aligned} \tan x &= \sin x \\ \frac{\sin x}{\cos x} &= \sin x && \text{(write } \tan x \text{ as } \frac{\sin x}{\cos x}\text{)} \\ \sin x &= \sin x \cos x && \text{(multiply by } \cos x\text{)} \\ 1 &= \cos x && \text{(cancel } \sin x\text{)} \\ x &= 0^\circ, 360^\circ \end{aligned}$$

Whilst the student did find two correct answers, there is another answer that they did not find.

(i) What did the student do wrong in their working? 1

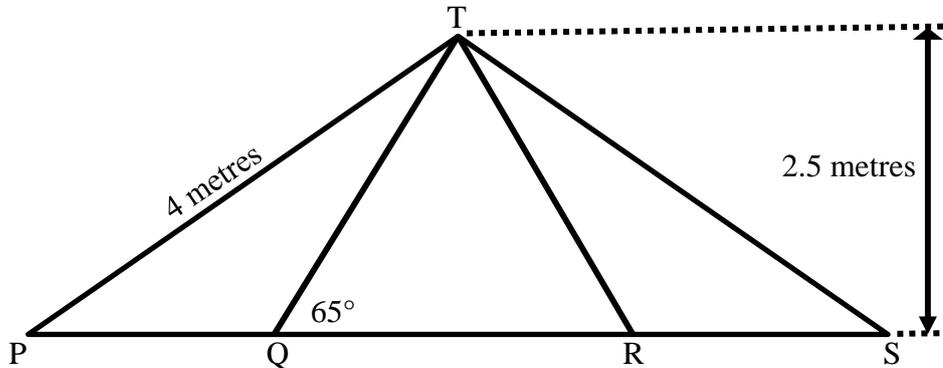
(ii) Find the missing answer. 1

(e) Solve, to the nearest degree where necessary, for $0^\circ \leq \theta \leq 360^\circ$ 3

$$\sin \theta \tan \theta + 2 \sin \theta = 3 \cos \theta.$$

Question 16 (12 marks) Use a separate answer sheet

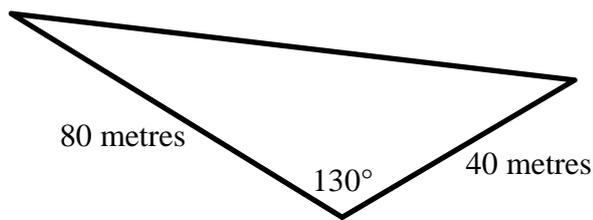
- (a) The diagram below represents a framework for supporting a roof. The outer beams TP and TS are each 4 metres in length, the inner beams TQ and TR are inclined at 65° to the horizontal and the height of the framework is 2.5 metres.



Calculate the;

- | | |
|--|---|
| (i) angle TPS, correct to the nearest degree | 1 |
| (ii) width PS, correct to the nearest centimetre. | 1 |
| (iii) length of TQ, correct to the nearest centimetre. | 1 |

- (b) A field is triangular, with two sides of 80 metres and 40 metres, enclosing an angle of 130° .

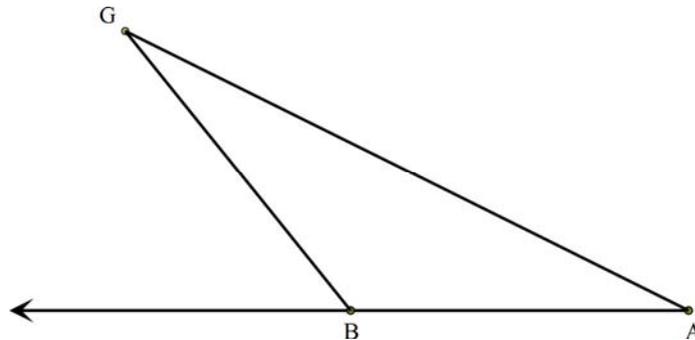


- | | |
|--|---|
| (i) Calculate the area of the field, correct to the nearest metre squared. | 1 |
| (ii) Without calculating its length, how do you know that the third side must be the largest side of the triangle? | 1 |
| (ii) Use the cosine rule to calculate the length of the third side, correct to the nearest metre. | 1 |

Question 16 continues on page 12

Question 16 (continued)

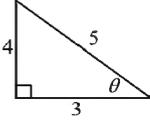
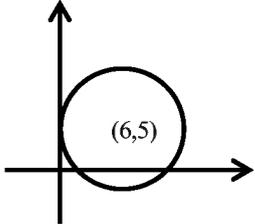
- (c) A and B are two points 1500 metres apart on a road running due west. A soldier at A observes that the bearing of an enemy's gun battery at G is 296° and heads towards B, where the bearing of the gun battery is 302° .



- (i) Copy this diagram onto your answer sheet, showing the given information. 1
- (ii) Explain why $\angle AGB = 6^\circ$ 1
- (iii) Show that $BG = \frac{1500\sin 26^\circ}{\sin 6^\circ}$ 1
- (iv) The range of the guns in the enemy's battery is 5 km. How far past B can the soldier travel before being in range of the enemy's guns? 3
Give your answer correct to the nearest metre.

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 11 MATHEMATICS HALF YEARLY ASSESSMENTS 2018 SOLUTIONS

Solution	Marks	Comments
SECTION I		
<p>1. B - $8x^3 + 27 = (2x)^3 + 3^3$ $= (2x + 3)((2x)^2 - (2x)(3) + 3^2)$ $= (2x + 3)(4x^2 - 6x + 9)$</p>	1	
<p>2. D – the only graph that satisfies the "vertical line test"</p>	1	
<p>3. A - $\frac{1 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + 3\sqrt{3} + 3}{4 - 3}$ $= 5 + 3\sqrt{3}$</p>	1	
<p>4. D – from the diagram; Angle of elevation is α Angle of depression is θ</p>	1	
<p>5. C - $\cos \theta < 0$ and $\sin \theta < 0 \Rightarrow$ quadrant 3, $\therefore \tan \theta > 0$ $\tan \theta = \frac{4}{3}$</p>	1	
<p>6. A - $\frac{x+5}{(x-3)(x+1)} - \frac{x-1}{x^2-x-2} = \frac{x+5}{(x-3)(x+1)} - \frac{x-1}{(x-2)(x+1)}$ $= \frac{(x+5)(x-2) - (x-1)(x-3)}{(x-3)(x-2)(x+1)}$ $= \frac{x^2 + 3x - 10 - x^2 + 4x - 3}{(x-3)(x-2)(x+1)}$ $= \frac{7x - 13}{(x-3)(x-2)(x+1)}$</p>	1	
<p>7. C - $\frac{\sin(360^\circ - A)}{\sin(90^\circ - A)} = \frac{-\sin A}{\cos A}$ $= -\tan A$</p>	1	
<p>8. B - $f(a) = g(2a)$ $3a^2 - 4 = (2a)^2 - 2(2a)$ $3a^2 - 4 = 4a^2 - 4a$ $a^2 - 4a + 4 = 0$ $(a - 2)^2 = 0$ $a = 2 \Rightarrow$ only one solution</p>	1	
<p>9. C - $x^2 + y^2 - 12x - 10y + k = 0$ $(x - 6)^2 + (y - 5)^2 = 61 - k \Rightarrow$ centre is (6,5) For exactly three intercepts the radius = 6 $61 - k = 36$ $k = 25$</p>	1	
<p>10. B - $\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}} = \frac{2^{r+s} \times 3^{r+s} \times 2^{2r-2s} \times 3^{r-s}}{2^{3r} \times 3^{2r+4s}}$ $= \frac{2^{3r-s} \times 3^{2r}}{2^{3r} \times 3^{2r+4s}}$ $= 2^{-s} \times 3^{-4s}$ Thus $s \leq 0$ in order for expression to be an integer</p>	1	

SECTION II
QUESTION 11

Solution	Marks	Comments
11(a) $\sqrt[4]{x^5} = x^{\frac{5}{4}}$	1	1 mark <ul style="list-style-type: none"> • Correct answer
11 (b) $\sqrt{\frac{4.81 \times 10^5}{7.36 \times 10^9}} = 0.00808413637\dots$ $= 0.0081 \text{ correct to two significant figures}$	2	2 marks <ul style="list-style-type: none"> • Correct solution 1 mark <ul style="list-style-type: none"> • Performs the correct calculation
11(c) $f(-1) = 7 - 2(-1)^2$ $= 7 - 2$ $= 5$	1	1 mark <ul style="list-style-type: none"> • Correct answer
11 (d) $\sqrt{75} - 2\sqrt{27} = 5\sqrt{3} - 6\sqrt{3}$ $= -\sqrt{3}$	2	2 marks <ul style="list-style-type: none"> • Correct solution 1 mark <ul style="list-style-type: none"> • Simplifies at least one surd correctly
11 (e) $(3x - 4)(x - 2)(x + 2)$ $= (3x - 4)(x^2 - 4)$ $= 3x^3 - 12x - 4x^2 + 16$ $= 3x^3 - 4x^2 - 12x + 16$	2	2 marks <ul style="list-style-type: none"> • Correct solution 1 mark <ul style="list-style-type: none"> • Performs a binomial product expansion
11 (f) (i) $2x^2 + 3x - 2$ $= (2x - 1)(x + 2)$	1	1 mark <ul style="list-style-type: none"> • Correct answer
11 (f) (ii) $x^3 + 5x^2 + x + 5$ $= x^2(x + 5) + 1(x + 5)$ $= (x + 5)(x^2 + 1)$	1	1 mark <ul style="list-style-type: none"> • Correct answer
11 (f) (iii) $4a^2(x^3 + 18ab^2) - (32a^5 + 9b^2x^3)$ $= x^3(4a^2 - 9b^2) - 8a^3(4a^2 - 9b^2)$ $= (4a^2 - 9b^2)(x^3 - 8a^3)$ $= (2a - 3b)(2a + 3b)(x - 2a)(x^2 + 2ax + 4a^2)$	2	2 marks <ul style="list-style-type: none"> • Correct solution 1 mark <ul style="list-style-type: none"> • Correctly uses $a^2 - b^2$ or $a^3 - b^3$ factorisation

QUESTION 12

12 (a) (i) $\frac{125a^3 - 8}{a^2 - 7a + 10} \times \frac{a - 5}{25a^2 - 4}$ $= \frac{(5a - 2)(25a^2 + 10a + 4)}{(a - 5)(a - 2)} \times \frac{a - 5}{(5a - 2)(5a + 2)}$ $= \frac{25a^2 + 10a + 4}{(a - 2)(5a + 2)}$	2	2 marks <ul style="list-style-type: none"> • Correct solution 1 mark <ul style="list-style-type: none"> • Factorises 2 out of the 3 non-linear expressions
12 (a) (ii) $\frac{2}{x^2 - 1} - \frac{1}{x^2 - x} + \frac{x - 1}{x^2 + x}$ $= \frac{2}{(x - 1)(x + 1)} - \frac{1}{x(x - 1)} + \frac{x - 1}{x(x + 1)}$ $= \frac{2x - (x + 1) + (x - 1)^2}{x(x + 1)(x - 1)}$ $= \frac{2x - x - 1 + x^2 - 2x + 1}{x(x + 1)(x - 1)}$ $= \frac{x^2 - x}{x(x + 1)(x - 1)}$ $= \frac{x(x - 1)}{x(x + 1)(x - 1)}$ $= \frac{1}{x + 1}$	3	3 marks <ul style="list-style-type: none"> • Correct solution 2 marks <ul style="list-style-type: none"> • Rewrites as a single fraction 1 mark <ul style="list-style-type: none"> • Finds the LCD

QUESTION 13...continued.

Solution	Marks	Comments
13 (f) (i) when $x = 2$ and $y = 1$ $(x + 2y)(2x - y) + (x - y)(3x + 4y) = (2 + 2)(4 - 1) + (2 - 1)(6 + 4)$ $= 12 + 10$ $= 22$	1	1 mark • Correct answer
13 (f) (ii) when $y = 1$ $(x + 2)(2x - 1) + (x - 1)(3x + 4) = 22$ $2x^2 + 3x - 2 + 3x^2 + x - 4 = 22$ $5x^2 + 4x - 28 = 0$ $(x - 2)(5x + 14) = 0$ $x = 2$ or $x = -\frac{14}{5}$ ∴ the only other possible value of x is $-\frac{14}{5}$	2	2 marks • Correct solution 1 mark • Establishes quadratic in terms of x

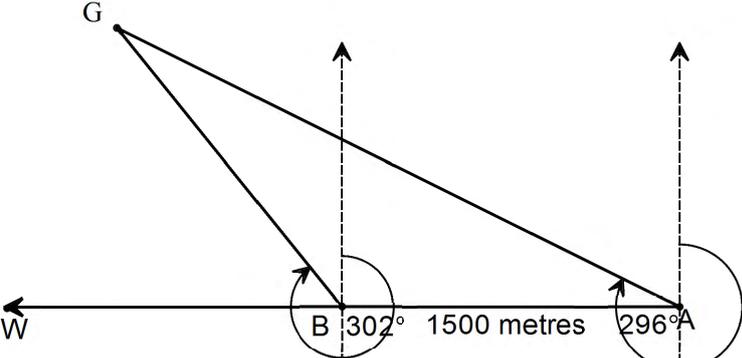
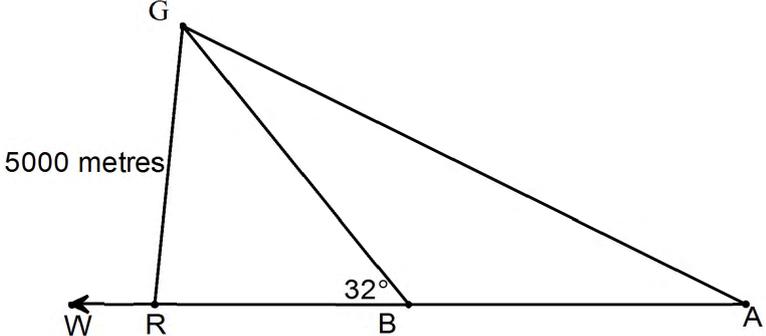
QUESTION 14

14 (a) (i) $f(x)$ is even if $f(-x) = f(x)$	1	1 mark • Correct condition
14 (a) (ii) some examples would include; • $y = x^2$ • $y = x $ • $y = \cos x$	1	1 mark • Correct example
14 (b) $f(2) + f(5) - f(-2) = 3(2) + (5^2 + 1) - 3(-2)$ $= 6 + 26 + 6$ $= 38$	2	2 marks • Correct solution 1 mark • finds at least two function values
14 (c) (i) $x^2 - 16 \geq 0$ $x \leq -4$ or $x \geq 4$	1	1 mark • Correct answer
14 (c) (ii) $y \geq 0$	1	1 mark • Correct answer
14 (d) (i) $P(0,1)$	1	1 mark • Correct answer
14 (d) (ii) $x^2 + y^2 = 2^2 + 4^2$ $= 20$ ∴ circle has equation $x^2 + y^2 = 20$	2	2 marks • Correct solution 1 mark • Recognises that a circle's equation is of the form $x^2 + y^2 = k$
14 (d) (iii) $(2,4) : 4 = a^2$ $a = 2$ ∴ exponential has the equation $y = 2^x$	1	1 mark • Correct answer
14 (d) (iv) $x \geq 0$ $x^2 + y^2 \leq 20$ $y \geq 2^x$	2	2 marks • Correct solution 1 mark • At least two correct inequations

QUESTION 15

15 (a) (i) $\cos 75^\circ = \sin(90^\circ - 75^\circ)$ $= \sin 15^\circ$ $= \frac{\sqrt{3} - 1}{2\sqrt{2}}$	1	1 mark • Correct solution
15 (a) (ii) $\operatorname{cosec} 15^\circ = \frac{1}{\sin 15^\circ}$ $= \frac{2\sqrt{2}}{\sqrt{3} - 1}$	1	1 mark • Correct solution
15 (a) (iii) $\sin 195^\circ = \sin(180^\circ + 15^\circ)$ $= -\sin 15^\circ$ $= -\frac{1 - \sqrt{3}}{2\sqrt{2}}$	1	1 mark • Correct solution

QUESTION 16...continued.

Solution	Marks	Comments
<p>16 (c) (i)</p> 	1	<p>1 mark</p> <ul style="list-style-type: none"> • Correct diagram with all information labelled
<p>16 (c) (ii)</p> $\begin{aligned} \angle GBW &= 302^\circ - 270^\circ = 32^\circ \\ \angle GAB &= 296^\circ - 270^\circ = 26^\circ \\ \angle GBW &= \angle AGB + \angle GAB \quad (\text{exterior } \angle, \triangle BAG) \\ 32^\circ &= \angle AGB + 26^\circ \\ \angle AGB &= 6^\circ \end{aligned}$	1	<p>1 mark</p> <ul style="list-style-type: none"> • Correct explanation <p><i>Note: formal geometric explanation not required</i></p>
<p>16 (c) (iii)</p> $\frac{BG}{\sin \angle GAB} = \frac{AB}{\sin \angle AGB}$ $\frac{BG}{\sin 26^\circ} = \frac{1500}{\sin 6^\circ}$ $BG = \frac{1500 \sin 26^\circ}{\sin 6^\circ}$	1	<p>1 mark</p> <ul style="list-style-type: none"> • Evidence of using sine rule in finding the correct expression, or similar merit.
<p>16 (c) (iv)</p>  $RG^2 = BG^2 + BR^2 - 2 \times BG \times BR \times \cos \angle RBG$ $BR^2 - (2 \times BG \times \cos \angle RBG) BR + (BG^2 - RG^2) = 0$ $BR = \frac{2 \times BG \times \cos 32^\circ \pm \sqrt{(2 \times BG \times \cos 32^\circ)^2 - 4(BG^2 - RG^2)}}{2}$ $= 1608.235614... \text{ or } 9061.388854$ <p>\therefore the soldier can travel 1608 metres</p>	3	<p>3 marks</p> <ul style="list-style-type: none"> • Correct solution <p>2 marks</p> <ul style="list-style-type: none"> • Establishes a quadratic equation in terms of BR or similar merit <p>1 mark</p> <ul style="list-style-type: none"> • Attempts to find RG using the cosine rule or similar merit. <p><i>Notes:</i></p> <ul style="list-style-type: none"> • <i>no rounding penalty</i> • <i>OK for approximate BG value (6290.695373...) to be used in working and calculations</i>