SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2014
YEAR 11
Yearly Examination

Mathematics

General Instruction

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise instructed

Total Marks – 75

- Attempt questions 1-6
- Hand up in 2 sections clearly marked A & B

Examiner: *T.Evans*

Question 1 **12 Marks**

a) Determine whether the point (-1, -8) lies on the line $2x-3y-23=0$ 2

b) Solve $2^x=3$ 2

c) Find the value of $\left(\frac{-27}{8}\right)^{\frac{-4}{3}}$ 1

d) Given that $\log_a b=2.75$ and $\log_a c=0.25$, find the value of: 2

i) $\log_a\left(\frac{b}{c}\right)$ 2

ii) $\log_a(bc)^2$

e) State whether the following are ODD, EVEN or neither: 1

i) $f(x)=3x+1$ 1

ii) $f(x)=x^4-x^2$

f) The first term of geometric sequence is 2 and the fourth term is 128. Find the common ratio. 1

Question 2 **13 Marks**

a) For the following sequence: 36, 24, 16,…… 2

Determine whether it is an Arithmetic or Geometric Progression, showing reasons.

b) Find $\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$ 2

c) Differentiate

i) $4x^2 - x + 1$ 1

ii) $\frac{3}{\sqrt[3]{x}}$ 2

iii) $\frac{3x^5 - 2x^3 + 5}{x^2}$ 2

iv) $\sqrt{3x^2 - 3}$ 2

v) $\frac{3x^2}{5 - 3x}$ 2

Question 3 **13 Marks**

a) i) Find all the x & y-intercepts for the curve with the equation $y = x^3 - 1$. 2

ii) Neatly sketch the curve $y = x^3 - 1$, showing the information from part i). 2

iii) Find the equation of the tangent to the curve $y = x^3 - 1$ at the point where $x = 1$. 2

b) If $f(x) = 15x^{-2} - 9x^3$, find the value of $f'(-1)$. 2

c) Differentiate $f(x) = x^2 - 3x$ by first principles using the definition 2

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

d) Simplify $\dfrac{\left(-2x^{\frac{1}{2}}y^{\frac{1}{3}}\right)^2}{\left(3x^{-1}y\right)^{\frac{1}{3}}} \times \dfrac{4x^{\frac{1}{3}}y^{-\frac{1}{3}}}{\left(x^{-1}y\right)^{\frac{1}{3}}}$ 3

Question 4 **12 Marks**

a) The equation $2x^2+5x-1=0$ has roots α and β. Find the values of:

i) $\alpha+\beta$ 1

ii) $\alpha\beta$ 1

iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ 2

iv) $\alpha^2+\beta^2$ 2

b) Consider the parabola $4y=x^2-4x$

i) Show, algebraically, how the parabola can be expressed in the form $(x-2)^2=4(y+1)$ 2

ii) Write down the coordinates of the vertex and the focus. 2

iii) Find the equation of the directrix. 2

Question Five **13 Marks**

a) i) Sketch $y = \log_2(x+2)$, showing the x & y-intersections and the asymptote. 2

ii) Hence, state the domain and range of $y = \log_2(x+2)$. 2

b) Find A, B & C such that $A(x-1)^2 + Bx + c = x^2$ 3

c) In a certain arithmetic series the 9th term is 78, and the 17th term is 50. Find:

i) The first term and the common difference 2

ii) The sum of the first forty terms 2

iii) The value of n for which the sum of the series is first negative. 2

Question 6 **12 Marks**

a) If $f(x)=x^2-x$ find the simplest expression for $f(x+h)$ 2

b) Express $0.42+0.0042+0.000042+.....$ as a simplified fraction 2

c) A function $y=f(x)$ is defined as follows:

$$f(x)=\begin{cases}-2, & x\le -1\\ x+1, & -1<x<2\\ x^2+1, & x\ge 2\end{cases}$$

i) Evaluate $f(-1)+f(2)$ 2

ii) Write an expression for $f(a^2+2)$ 2

d) Solve for x

i) $4^x=12(2)^x-32$ 2

ii) $2\log_5 3=\log_5 x-\log_5 6$ 2

End of Examination

2014 Year 11 Mathematics Yearly - Solutions

Question 1

(a) $2x - 3y - 23 = 0 \quad (-1, -8)$

$2(-1) - 3(-8) - 23 = -1 \quad (\neq 0)$

$(-1, -8)$ not on line

(b) $2^x = 3$

$\log 2^x = \log 3$

$x \log 2 = \log 3$

$x = \dfrac{\log 3}{\log 2} \approx 1.585 \; (3 \text{ d.p.})$

(c) $\left(-\dfrac{27}{8}\right)^{-\frac{4}{3}} = \left(-\dfrac{3}{2}\right)^{-4} = \dfrac{16}{81}$

(d) (i) $\log_a\left(\dfrac{b}{c}\right) = \log_a b - \log_a c = 2.5$

(ii) $\log_a (bc)^2 = 2(\log_a b + \log_a c)$

$= 6$

(e) (i) $f(x) = (3x+1) \quad f(-x) = -3x+1$

neither

(ii) $f(x) = x^4 - x^2, \quad f(-x) = x^4 - x^2$

even.

(f) $a = 2, \quad T_4 = 128$

$ar^3 = 128$

$r^3 = 64$

$r = 4$

Question 2

(a) $\frac{24}{36} = \frac{16}{24} = \frac{2}{3}$

$\therefore$ Geometric

(b) $\lim\limits_{x \to} = \frac{x^2 - x - 12}{x - 4} = \lim\limits_{x \to 4} \frac{(x-4)(x+3)}{x-4}$

$= \lim\limits_{x \to 4} x + 3 = 7.$

(c)(i) $f(x) = 4x^2 - x + 1 \qquad f'x = 8x - 1$

(ii) $f(x) = 3x^{-\frac{1}{3}}, \qquad f'(x) = -x^{-\frac{4}{3}}$

$= -\frac{1}{\sqrt[3]{x^4}}$

(iii) $f(x) = \frac{3x^5 - 2x^3 + 5}{x^2} = 3x^3 - 2x + 5x^{-2}$

$f'(x) = 9x^2 - 2 - \frac{10}{x^3}$

(iv) $f(x) = (3x^2 - 3)^{\frac{1}{2}} \quad f'(x) = \frac{1}{2}(3x^2 - 3)^{-\frac{1}{2}} \times 6x$

$= \frac{3x}{\sqrt{3x^2 - 3}}$

(v) $f(x) = \frac{3x^2}{5 - 3x} = \frac{u}{v} \qquad f'(x) = \frac{vu' - uv'}{v^2}$

$f'(x) = \frac{(5-3x) \times 6x - 3x^2(-3)}{(5-3x)^2}$

$= \frac{30x - 18x^2 + 9x^2}{(5-3x)^2}$

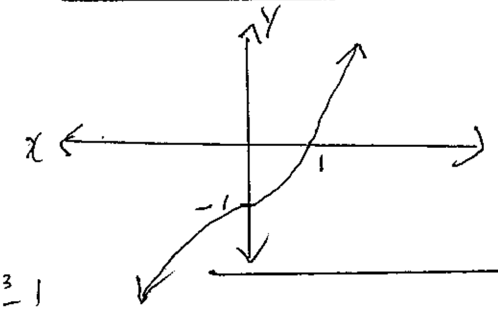
$= \frac{30x - 9x^2}{(5 - 3x)^2}$

Question 3

(a)(i) $y = x^3 - 1$

$y = 0 \quad x = 1 \quad , (1,0) \quad , x = 0 \quad y = -1 \quad (0,-1)$

(b)(ii)



(iii) $y = x^3 - 1$

$y' = 3x^2 \quad , x = 1 \quad y' = 3$

$y - y_1 = m(x - x_1) \quad (1,0)$

$y - 0 = 3(x - 1) \quad , \quad y = 3x - 3$

(b) $f(x) = 15x^{-2} - 9x^3 \quad , \quad f'(x) = -30x^{-3} - 27x^2$

$f'(-1) = -30(-1) - 27(1) = 3$

(c) $\lim_{h\to 0} \left(\frac{(x+h)^2 - (x^2 - 3x)}{h}\right) = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3(x+h) - (x^2 - 3x)}{h}$

$= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h}$

$= \lim_{h\to} 2x + h - 3 = 2x - 3$

(d) $\frac{\left(-2x^{\frac{1}{2}}y^{\frac{1}{3}}\right)^2}{\left(3x^{-1}y\right)^{\frac{1}{3}}} \times \frac{4x^{\frac{1}{3}}y^{\frac{1}{3}}}{\left(x^{-1}y\right)^{\frac{1}{3}}}$

$= \frac{16x^{\frac{4}{3}}y}{3^{\frac{1}{3}}x^{-\frac{2}{3}}y^{\frac{2}{3}}}$

$= \frac{16}{\sqrt[3]{3}}x^2y^{\frac{1}{3}}$

4) a) i) $\alpha+\beta=-\frac{b}{a}$

$=-\frac{5}{2}$

ii) $\alpha\beta=\frac{c}{a}$

$=-\frac{1}{2}$

iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{\left(-\frac{5}{2}\right)}{\left(-\frac{1}{2}\right)}$

$=5$

iv) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

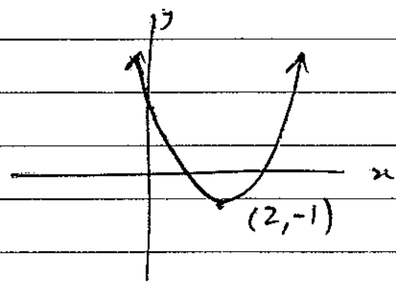
$=\left(-\frac{5}{2}\right)^2-2\left(-\frac{1}{2}\right)$

$=\frac{29}{4}$

b) i) $x^2-4x=4y$

$x^2-4x+4=4y+4$

$(x-2)^2=4(y+1)$



ii) In the form $(x-h)^2=4a(y-k)$

$4a=4$

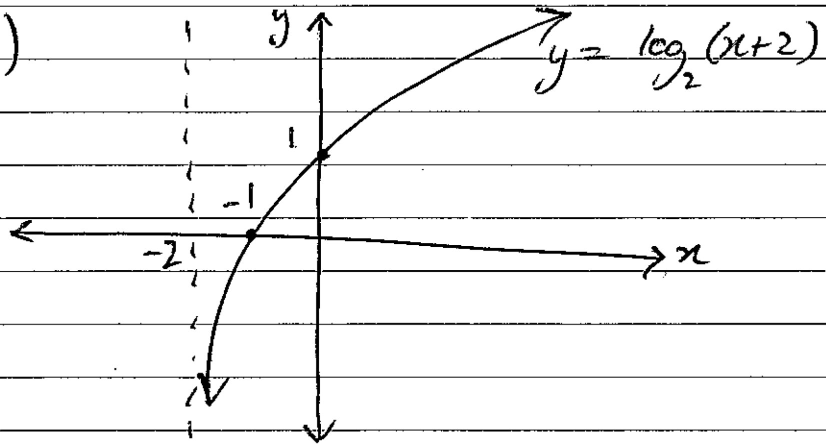
$a=1$

Vertex $(2,-1)$

Focus $(2,0)$

iii) $y=-2$

5) a) i)



ii) Domain: $x > -2$

Range: all real y.

b) $A(x-1)^2 + Bx + C \equiv x^2$

equate coefficients of x^2

$\underline{A = 1}$

let $x = 0$

$A((0)-1)^2 + \cancel{B(0)} + C = (0)^2$

$A + C = 0$

$C = -A$

$\underline{C = -1}$

let $x = 1$

$A(\cancel{(1)-1})^2 + B(1) + C = (1)^2$

$B + C = 1$

$B + (-1) = 1$

$\underline{B = 2}$

c) $T_9 = 78$

$T_{17} = 50$

$T_n = a + (n-1)d$

$a + 8d = 78$ —— (1)

$a + 16d = 50$ —— (2)

(2) − (1)

$8d = -28$ —— (3)

$\underline{d = -3.5}$

sub ③ into ①

$a + (-28) = 78$

$\underline{a = 106}$

ii) $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

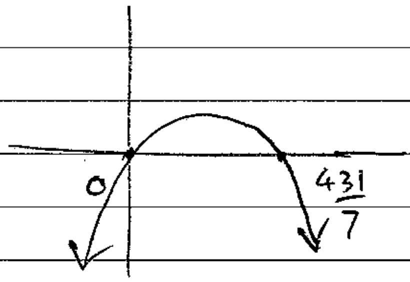
$S_{40} = \frac{(40)}{2}\left(2(106) + ((40)-1)(-3.5)\right)$

$= 1510$

iii) $S_n < 0$

$\frac{n}{2}\left(2(106) + (n-1)(-3.5)\right) < 0$

$\frac{n}{2}\left(215.5 - 3.5n\right) < 0$



since n is a positive integer

$n > \frac{431}{7}$

$n > 61.\dot{5}7142\dot{8}$

$\underline{n = 62}$

6) a) $f(x) = x^2 - x$

$$f(x+h) = (x+h)^2 - (x+h)$$
$$= x^2 + 2xh + h^2 - x - h$$

OR

$$(x+h)(x+h-1)$$

b) $0.42 + 0.0042 + 0.000042 + \ldots$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{\left(\frac{42}{100}\right)}{1 - \left(\frac{1}{100}\right)}$$

$$= \frac{14}{33}$$

OR

$0.\dot{4}\dot{2} = 0.42 + 0.0042 + 0.000042 + \ldots$

let $x = 0.4242\ldots$

$100x = 42.4242\ldots$

$99x = 42$

$x = \frac{42}{99}$

$x = \frac{14}{33}$

c) i) $f(-1) + f(2) = [-2] + [(2)^2 + 1]$

$$= 3$$

ii) $f(a^2+2) = (a^2+2)^2 + 1$ Note: $a^2 + 2 \geq 2$

$$= a^4 + 2a^2 + 4 + 1$$

$$= a^4 + 2a^2 + 5$$

d) i) $4^x = 12(2)^x - 32$

$(2^2)^x - 12(2)^x + 32 = 0$

$(2^x)^2 - 12(2)^x + 32 = 0$

let $y = 2^x$

$y^2 - 12y + 32 = 0$ $\quad\quad$ $\times \mid 32$; $+ \mid -12$

$(y-8)(y-4) = 0$

$y = 8$ or $y = 4$

$2^x = 8$ $\quad\quad$ $2^x = 4$

$2^x = 2^3$ $\quad\quad$ $2^x = 2^2$

$\therefore x = 3, 2$

ii) $2\log_5 3 = \log_5 x - \log_5 6$

$\log_5 3^2 = \log_5 \left(\frac{x}{6}\right)$

$\frac{x}{6} = 9$

$x = 54$