Ryde SECONDARY COLLEGE
INSPIRE ACHIEVE SUCCEED

Full Name: ____________________

Teacher: ____________________

2017

YEAR 11

PRELIMINARY EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided

Total marks - 70

Section I

10 marks

Use the multiple choice answer sheet

Section II

60 marks

Start a new answer booklet for each of these questions. No answers to be written on question booklet.

Section I (10 marks)

Use the multiple-choice answer sheet for Questions 1-10

1. Simplify $\dfrac{x^3 + x}{x}$

(A) $x^2 + 1$ (B) $x^2 + x$ (C) $x^3 + 1$ (D) x^3

2. $3^{500} \times 3^{20} =$

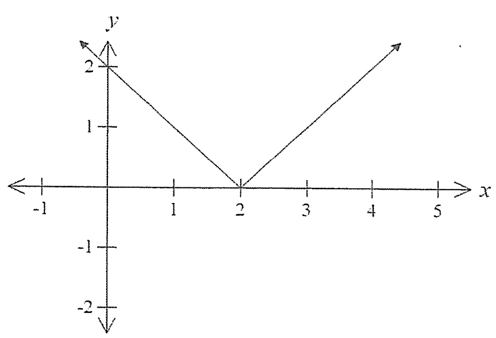
(A) 9^{520} (B) $9^{10\,000}$ (C) $3^{10\,000}$ (D) 3^{520}

3. Which of these is a function?

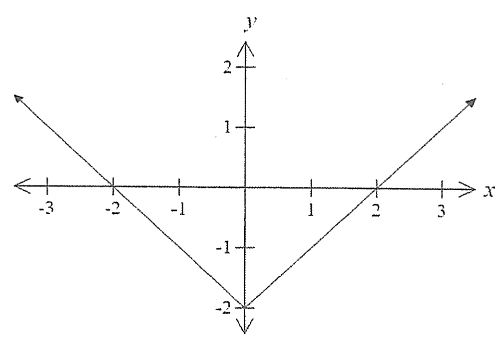
(A) $y^2 = x$ (B) $y = 1$ (C) $x^2 + y^2 = 4$ (D) $x = 2$

4. Which graph best represents $y = |x| - 2$?

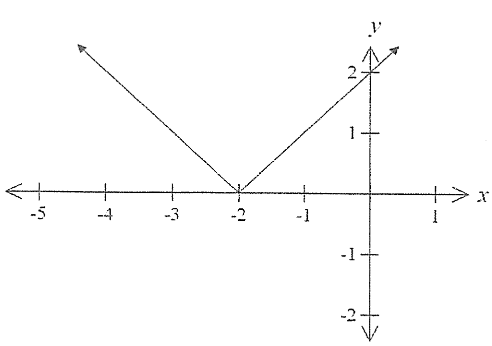
(A)



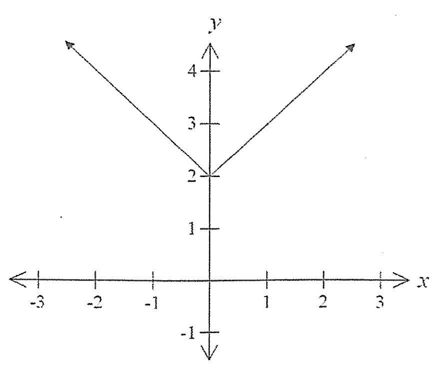
(B)



(C)



(D)



5. If $y = 2(3x - 1)^{-4}$, which of the following is the correct expression for $\frac{dy}{dx}$?

(A) $\frac{-24}{(3x-1)^5}$ (B) $\frac{1}{24(3x-1)^5}$ (C) $\frac{1}{8(3x-1)^5}$ (D) $\frac{8}{(3x-1)^5}$

6. Evaluate $\sqrt[4]{\frac{43.52 \times 6.23}{4.3^2}}$, correct to two significant figures.

(A) 1.9 (B) 1.95 (C) 2.0 (D) 3.8

7. The quadratic equation $x^2 - 3x - 9 = 0$ has roots α and β.

What is the value of $\frac{1}{\alpha} + \frac{1}{\beta}$?

(A) 3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) -3

8. For what values of r does the equation $rx^2 + rx + 1$ have two unequal real roots?

(A) $r < 0$ (B) $r > 4$ (C) $r < 0$, $r > 4$ (D) $0 < r < 4$

9. For what values of p is the line $px + 2y = 0$ parallel to the line $8x + py = 0$?

(A) $p = 4$ (B) $p = -4$ (C) $p = \pm 4$ (D) $p = \pm 16$

10. Simplify $\text{cosec}^2 x \sec^2 x - \text{cosec}^2 x$

A) $\text{cosec}^2 x$ B) $\sin^2 x$ C) $\tan^2 x$ D) $\sec^2 x$

Section II

Question 11 (15 marks) **Start a new answer booklet** **Marks**

a) Solve the equation $2\cos x = \sqrt{2}$ for $0° \le x \le 360°$ **3**

b) Consider the function $y = \sqrt{9 - x^2}$.

i) Sketch the graph of the function. **2**

ii) State the range of the function. **1**

c) Solve the inequality $|5x - 2| \ge 4$. **2**

d) Differentiate with respect to x

i) $y = \dfrac{x}{2x^2 + 1}$. **2**

ii) $y = x(2x+1)^2$ **2**

e) Find the point of intersection of the lines $4x - 3y - 3 = 0$ and $x - 3y + 15 = 0$ **2**

f) **1**

Question 12 (15 marks) **Start a new answer booklet** **Marks**

a) Solve the equation $x(x-5)=-6$. **2**

b) Find the radius of the circle with equation $x^2+y^2-4y-12=0$. **1**

c) Find the gradient of the tangent to curve $y=\sqrt{x^2+4}$ at the point (0, 2). **2**

d) **3**

e) Consider the function $f(x)=px^2-2x$. **2**

Given that $f(1)=f(-2)$, find the value of p.

f)

Question 13 (15 marks) **Start a new answer booklet** **Marks**

a) Find a quadratic equation with roots $\sqrt{2}$ and $-\sqrt{2}$. **1**

b) **3**

c) Solve $2\log_7 4 = \log_7 2x - \log_7 3$. **2**

d) The tangent to the parabola $y = \frac{1}{4}x^2 - 4x$ at a point P has a gradient of -6.
Find the coordinates of the point P. **3**

e) Find the equation of the normal to the curve $y = \frac{3}{1+2\sqrt{x}}$ at the point where $x = 1$. **4**

f) Show that $\frac{\operatorname{cosec}^2 x - 1}{\cos^2 x} = \operatorname{cosec}^2 x$ **2**

Question 14 (15 marks) **Start a new answer booklet** **Marks**

a) Find the values of the constant m such that $y = mx$ is tangent to the parabola with equation $y = 4x^2 + 1$. **3**

b)

2

2

1

c) Sketch the curve $y = 2\cos 2x$ for $0 \le x \le 360^\circ$. **2**

d) Show that the quadratic equation $x^2 - mx - 1 = 0$ has two real unequal roots for all values of m. **2**

e) **3**

End of paper

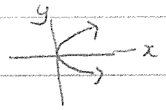
2017 Practice Advanced

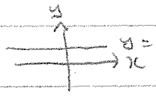
1. $\frac{x^3+x}{x} = \frac{x(x^2+1)}{x}$

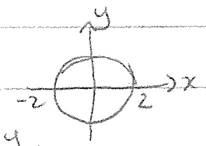
$= x^2+1 \quad (x \neq 0) \quad (A)$

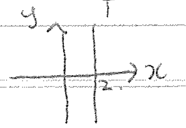
2. $3^{500} \times 3^{20} = 3^{520} \quad (D)$

Multiply $\Rightarrow$ add the indices

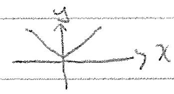
3. (A) $y^2 = x$  (NO)

(B) $y = 1$  (YES)

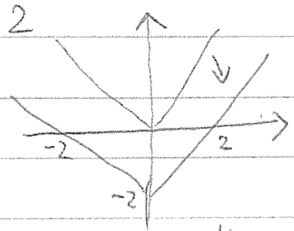
(C) $x^2+y^2=4$  (NO)

(D) $x = 2$  (NO)

Must be 1 y for each x! (B)

4. $y = |x|$ 

$y = |x| - 2 \Rightarrow$ shift down by 2 (B)



5. $y = 2(3x-1)^{-4}$

$\frac{dy}{dx} = 2 \times (-4) \times (3) \times (3x-1)^{-5}$

$= -24(3x-1)^{-5}$

$= \frac{-24}{(3x-1)^5} \quad (A)$

6 $\sqrt{\frac{43.52 \times 6.23}{4.3^2}}$

$= 3.829305461$ (calc)

$= 3.8$ to 2 sig. figs (D)

7. $x^2 - 3x - 9 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

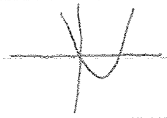
$= \frac{(-3)}{(-9)}$

$= 1/3 \quad (C)$

8. $rx^2 + rx + 1$

2 unequal real roots $\Rightarrow \Delta > 0$

$r^2 - 4r > 0$

$r(r-4) > 0$ 

$\therefore r < 0$ or $r > 4 \quad (C)$

9 $px + 2y = 0 \qquad 8x + py = 0$

$2y = -px \qquad py = -8x$

$y = \frac{-p}{2}x \qquad y = \frac{-8}{p}x$

$\therefore \frac{-p}{2} = \frac{-8}{p}$

$p^2 = 16$

$p = \pm 4 \quad (C)$

10, $\operatorname{cosec}^2 x \sec^2 x - \operatorname{cosec}^2 x$

$= \operatorname{cosec}^2 x (\sec^2 x - 1)$

Now $\cos^2\theta + \sin^2\theta = 1$

$(\div \sin^2\theta)$ $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$(\div \cos^2\theta)$ $1 + \tan^2\theta = \sec^2\theta$

$\therefore$ LHS $= \operatorname{cosec}^2 x . \tan^2 x$

$= \dfrac{1}{\sin^2 x} . \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

$=$ LHS.

Question 11

a) $2\cos x = \sqrt{2}$ $\quad 0 \le x \le 360^\circ$

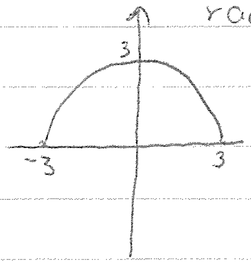
$\cos x = \dfrac{\sqrt{2}}{2}$ S | ✓ A, T | ✓ C

$x = 45^\circ, 360 - 45^\circ$

$= 45^\circ, 315^\circ$

b) $y = \sqrt{9 - x^2}$

i) semicircle centre (0,0) radius = 3



ii) Range : [-3, 3]

c) $|5x - 2| \geq 4$

$5x - 2 \geq 4$ $\qquad -(5x-2) \geq 4$

$5x \geq 6$ $\qquad -5x + 2 \geq 4$

$x \geq \dfrac{6}{5}$ $\qquad -5x \geq 2$

$x \leq \dfrac{-2}{5}$

$\therefore x \leq \dfrac{-2}{5}$ or $x \geq \dfrac{6}{5}$

Always 2 processes, +LHS, -LHS and check your answers! (by substituting a value)

d) i) $y = \dfrac{x}{2x^2+1}$ $\qquad u = x \quad u' = 1$, $v = 2x^2+1 \quad v' = 4x$

$\dfrac{dy}{dx} = \dfrac{(2x^2+1).1 - x.4x}{(2x^2+1)^2}$

$= \dfrac{2x^2 + 1 - 4x^2}{(2x^2+1)^2}$

$= \dfrac{-2x^2+1}{(2x^2+1)^2}$

Never expand the denominator

ii) $y = x(2x+1)^2$ $\qquad u = x \quad u' = 1$, $v = (2x+1)^2$, $v' = 4(2x+1)$

$\dfrac{dy}{dx} = (2x+1)^2 \times 1 + x \times 4(2x+1)$

$= (2x+1)[2x+1+4x]$

$= (2x+1)(6x+1)$

Check reference sheet.

Note: retained factored form to make simplifying easy

e) $4x - 3y - 3 = 0$ ... ①

$x - 3y + 15 = 0$ ... ②

For point of intersection, solve simultaneously

①−② $3x - 18 = 0$

$3x = 18$

$x = 6$

sub into ①

$24 - 3y + 3 = 0$

$3y = 27$

$y = 9$

$\therefore x = 6, y = 9$ ie $(6, 9)$

Answer the question. Point of intersection means coordinate

QUESTION 12

a) $x(x-5) = -6$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\therefore x = 3, 2$

b) $x^2 + y^2 - 4y - 12 = 0$

Complete the square on y:

$x^2 + \left[(y-2)^2 - 4\right] - 12 = 0$

$x^2 + (y-2)^2 - 16 = 0$

$x^2 + (y-2)^2 = 16$

$\therefore$ radius is 4 $(\sqrt{16})$

{centre is $(0, 2)$}

c) $y = \sqrt{x^2+4}$

$= (x^2+4)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2} \times 2x\,(x^2+4)^{-1/2}$

$= \frac{x}{\sqrt{x^2+4}}$

At $(0, 2)$, gradient of tangent

$m = \frac{dy}{dx} = \frac{0}{\sqrt{0+4}}$

$= 0$

For the gradient, always substitute the x-value into $\frac{dy}{dx}$

e) $f(x) = px^2 - 2x$

Given $f(1) = f(-2)$

$f(1) = p - 2$

$f(-2) = 4p + 4$

$\therefore 4p + 4 = p - 2$

$3p = -6$

$p = -2$

QUESTION 13

a) $\alpha = \sqrt{2}$ $\beta = -\sqrt{2}$

$q(x) = (x-\alpha)(x-\beta)$

$= x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - (\sqrt{2}-\sqrt{2})x + \sqrt{2}.(-\sqrt{2})$

$= x^2 - 2$

This is monic. Could be $a(x^2-2)$ for $a \neq 0$

c) $2\log_7 4 = \log_7 2x - \log_7 3$

$\log_7 4^2 = \log_7 \frac{2x}{3}$

$4^2 = \frac{2x}{3}$

$x = \frac{4^2 \times 3}{2}$

$= 24$

Log laws: $\log_a x^n = n \log_a x$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

d) $y = \frac{1}{4}x^2 - 4x$

Gradient function:

$\frac{dy}{dx} = 2 \times \frac{1}{4} \times x - 4$

$= \frac{1}{2}x - 4$

If $m = -6$

$\frac{1}{2}x - 4 = -6$

$\frac{1}{2}x = -2$

$x = -4$

At $x = -4$

$y = \frac{1}{4}(-4)^2 - 4(-4)$

$= 4 + 16$

$= 20$

$\therefore$ $P = (-4, 20)$

e) $y = \frac{3}{1 + 2\sqrt{x}}$

$= 3(1 + 2x^{1/2})^{-1}$

$\frac{dy}{dx} = 3 \times (-1) \times \left(\frac{2x^{-1/2}}{2}\right)(1 + 2x^{1/2})^{-2}$

$= \frac{-3}{\sqrt{x}(1 + 2\sqrt{x})^2}$

At $x = 1$

$\frac{dy}{dx} = \frac{-3}{1 \times 3^2}$

$= \frac{-3}{9}$

$= -\frac{1}{3}$

Nasty chain rule. Index form first. Could use $u = 1 + 2x^{1/2}$

$\therefore \frac{du}{dx} = 2 \cdot \frac{1}{2} \cdot x^{-1/2}$

$= x^{-1/2}$

$\therefore y = 3u^{-1}$ $\frac{dy}{du} = \frac{-3}{u^2}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{-3}{(1 + 2\sqrt{x})^2} \cdot \frac{1}{\sqrt{x}}$

$= \frac{-3}{\sqrt{x}(1 + 2\sqrt{x})^2}$

f) $\dfrac{\operatorname{cosec}^2 x - 1}{\cos^2 x} = \operatorname{cosec}^2 x$

NB:

Pythagoras:

$\cos^2 x + \sin^2 x = 1$

$\div \sin^2 x \quad \cot^2 x + 1 = \operatorname{cosec}^2 x$

$\text{LHS} = \dfrac{\operatorname{cosec}^2 x - 1}{\cos^2 x}$

$= \dfrac{\cot^2 x}{\cos^2 x}$ by Pyth.

$= \dfrac{\cos^2 x}{\sin^2 x} \cdot \dfrac{1}{\cos^2 x}$

$= \dfrac{1}{\sin^2 x}$

$= \operatorname{cosec}^2 x = \text{RHS}$

as required

d) $x^2 - mx - 1 = 0$

Two real unequal roots $\Rightarrow \Delta > 0$

$\Delta = b^2 - 4ac$

$= m^2 - 4(-1)$

$= m^2 + 4$

This is a parabola with vertex $(0,4)$ and is concave up.

$\therefore \Delta \geq 4$ for all m

as required

QUESTION 14

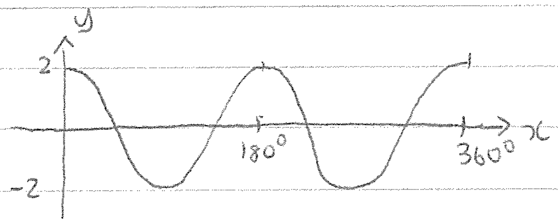
a) $y = mx \quad y = 4x^2 + 1$

For parabola, tangent gradient function:

$\dfrac{dy}{dx} = 8x$

$\therefore m = 8$

c)



Period halved $\quad y = A\cos\dfrac{x}{n}$

Amplitude doubled