

2020

Trial Higher School Examination

Mathematics Advanced

General Instructions

- Reading time 10 minutes
- · Working time 3 hours
- · Write using black pen
- Calculators approved by NESA may be used
- · A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Section I – 10 marks (pages 2-4)

Total marks: 100

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5-24)

- Attempt Questions 11-32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt questions 1 - 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

- 1. What is the value of $\csc \frac{\pi}{3}$ to three significant figures?
 - (A) 1.00
 - (B) 1.15
 - (C) 1.41
 - (D) 2.00
- 2. What is the value of c for which the circle $(x-3)^2 + (y-2)^2 = c$ touches the x axis?
 - (A) 2
 - (B) 3
 - (C) 4
 - (D) 9
- 3. What is the equation of the tangent to $y = x^2 3$ at x = -1?
 - (A) y = -2x 4
 - (B) y = 2x 4
 - (C) $y = \frac{x}{2} \frac{3}{2}$
 - (D) $y = -\frac{x}{2} \frac{3}{2}$

- 4. Which statement is true for an ungrouped data set with no outliers?
 - (A) The largest possible range is 2 times the interquartile range.
 - (B) The largest possible range is 3 times the interquartile range.
 - (C) The largest possible range is 4 times the interquartile range.
 - (D) The largest possible range is 5 times the interquartile range.
- 5. Which one of the following is the set of all solutions to $2x^2 5x + 2 \ge 0$?
 - (A) $\left[\frac{1}{2}, 2\right]$
 - (B) $\left(\frac{1}{2},2\right)$
 - (C) $\left(-\infty, \frac{1}{2}\right) \cup \left(2, \infty\right)$
 - (D) $\left(-\infty,\frac{1}{2}\right]\cup\left[2,\infty\right)$
- 6. The graph of y = f(x) has a stationary point at (2,-3).

Which one of the following is a guaranteed stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

- (A) (1,-2)
- (B) (1,2)
- (C) (4,-2)
- (D) (4,2)

- 7. What is the period and amplitude for the curve $y = \sin \pi x$?
 - (A) Amplitude = 1; Period = 2
 - (B) Amplitude = π ; Period = 2
 - (C) Amplitude = 1; Period = 2π
 - (D) Amplitude = π ; Period = 2π
- 8. If the z scores on an examination are normally distributed and P(z < N) = 0.6 for some number N, what is the value of P(-N < z < N)?
 - (A) 0.1
 - (B) 0.2
 - (C) 0.3
 - (D) 0.4
- 9. Which one of the following equations is NOT correct?

(A)
$$\int x(x^2 - 1)^2 dx = \frac{(x^2 - 1)^3}{6} + c$$

(B)
$$\int_{-3}^{3} \sqrt{9 - x^2} dx = \frac{9\pi}{2}$$

(C)
$$\int_{-1}^{1} 3^{x} dx = \frac{1}{\ln 3} \left(3 - \frac{1}{3} \right)$$

(D)
$$\int_{-5}^{5} 4x^4 - x^3 + \cos x \, dx = 0$$

- 10. Consider the series $\sqrt{5} + \sqrt{45} + \sqrt{125} + \dots + z = 225\sqrt{5}$, the value of z is:
 - (A) $25\sqrt{5}$
 - (B) $29\sqrt{5}$
 - (C) $30\sqrt{5}$
 - (D) $35\sqrt{5}$

Section II

90 marks

Attempt all questions

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks)

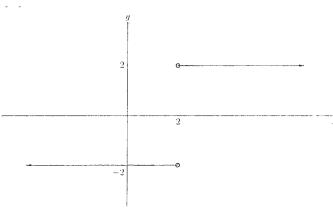
Marks

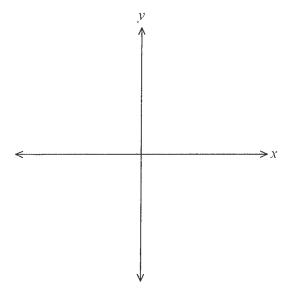
What angle does the line 2x + 3y + 6 = 0 make with the positive *x*-axis? Round to the nearest minute.

2

Question 12 (2 marks)

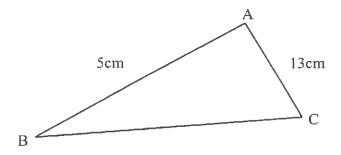
Sketch a possible function which could have the gradient function as graphed below.





3

In triangle *ABC*, the length of *AB*=5cm, *AC*=13cm and $\cos \langle BAC = \frac{1}{8} \rangle$



| (a) | Find the exact value of $\sin(BAC)$ | |
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| (b) | Find the area of triangle ABC | 1 |
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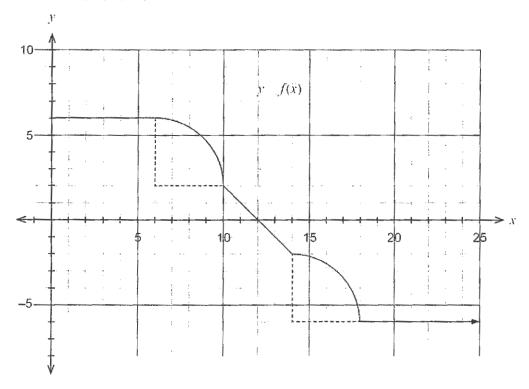
Question 14 (3 marks)

| Solve $2\log x = \log(5x+6)$ |
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| | $1 - 2\cos^2 x$ = 1 for $0 \le x \le 2\pi$ | M |
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| | stion 16 (5 marks) rentiate the following expressions. | M |
| 1) | $\log_5(\tan x)$ | |
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| b) | $\frac{2^x}{e^x}$ | |
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2

Consider the graph y = f(x). Both arcs have a radius of four units.



Using the graph of y = f(x), $x \ge 0$, evaluate exactly the following integrals.

| (a) | $\int_{0}^{12} f(x)dx$ | 3 |
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| (b) | $\int_{0}^{18} f(x)dx$ |
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The discrete random variable X has probability distribution shown in the table below

| x | -1 | 0 | 1 | 2 | 3 |
|--------|----|---|-----|------|------|
| P(X=x) | a | b | 0.2 | 0.15 | 0.13 |

and E(X) = 0.55

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| Que | stion 19 (2 marks) | Marks |
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| devi | length of steel rods produced by a machine is normally distributed with a standard ation of 3 mm. It is found that 2.5% of all rods are less than 25 mm long. Find the n length of rods produced by the machine. | 2 |
| | | |
| Que | stion 20 (8 marks) | |
| Cons | sider the function $f(x) = x^3 + 6x^2 + 9x + 4$ in the domain $-4 \le x \le 1$ | |
| (a) | Find the coordinates of any stationary points and determine their nature. | 3 |
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| (b) | Determine the coordinates of its point(s) of inflexion. | 2 |
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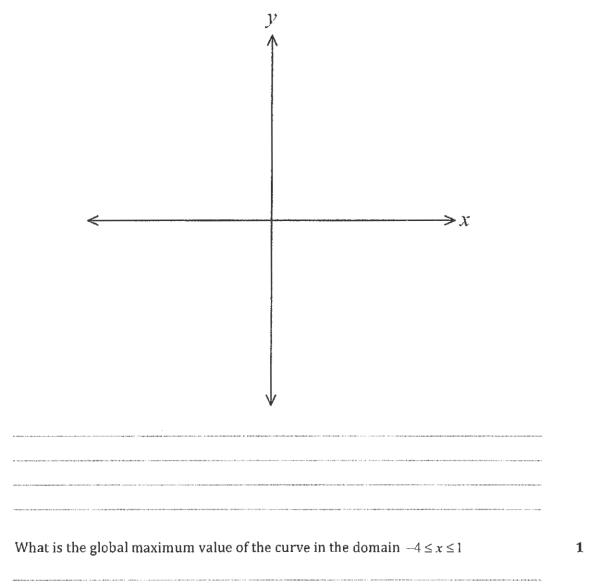
Question 20 continued on next page

Question 20 continued

Marks

(c) Draw a sketch of the curve y = f(x) in the domain $-4 \le x \le 1$ clearly showing all essential features.

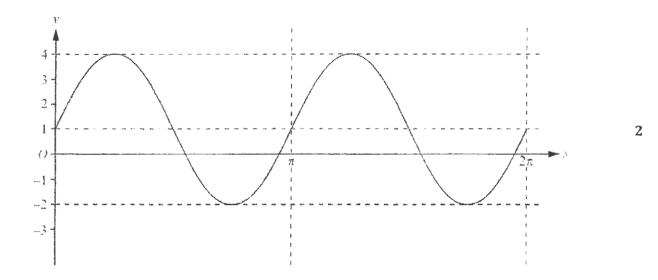
Marks 2



(d)

Question 21 (2 marks)

The diagram shows the graph of $y = a \sin(bx) + c$ for $0 \le x \le 2\pi$, where a, b and c are positive integers.



| Find the values of a , b and c . |
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Question 22 (5 marks)

Marks

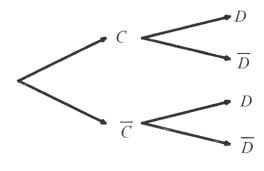
A pet ownership survey resulted in the following results:

$$P(C) = \frac{3}{7}$$
, $P(D|\overline{C}) = \frac{2}{5}$, and $P(\overline{D}|C) = \frac{3}{4}$.

Where C is the event that "a person has a cat" and D is the event that "a person has a dog"

(a) Complete the probability tree by marking a probability on each branch.

2



(b) If one person is chosen at random, find the probability that the person has:

| i) a cat and a dog | 1 |
|--------------------|---|
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ii) at least one pet (cat or dog)

Question 23 (5 marks)

Marks

3

The function f is defined by $f(x) = 2 + \sqrt{x-3}$ for $x \ge 3$.

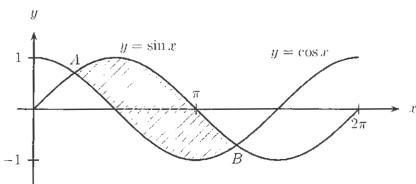
The function g is defined by $g(x) = \frac{12}{x} + 2$ for x > 0

| (a) | Write the domain and range of the function f using interval notation. | |
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| (b) | Write an expression for the composite function $h(x) = g(f(x))$ and hence find a value for $g(f(12))$ | 2 |
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Question 24 (5 marks)

The diagram shows the graphs $y = \sin x$ and $y = \cos x$, $0 \le x \le 2\pi$. The graphs intersect at A and B.



| (a) | Show that A has coordinates $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$. | 2 |
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| | $(4,\sqrt{2})$ | |
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Question 24 continued on next page

Question 24 continued

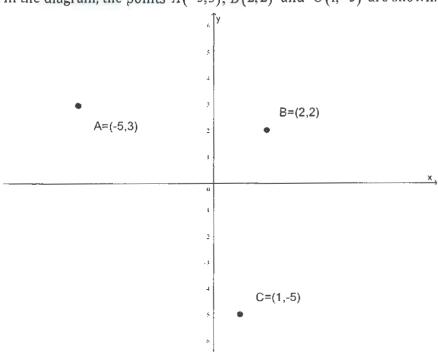
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(c)

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In the diagram, the points A(-5,3), B(2,2) and C(1,-5) are shown.



| (a) | Calculate the gradient of AC. | 1 |
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| (b) | Find the coordinates of X , the midpoint of AC . |
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| Find the coordinates of D if X is also the midpoint of BD . | |
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| Ouestion | 26 | (5 | marks) | |
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Marks

A particle is moving in a straight line. Its velocity for $t \ge 0$ is given by $v = \frac{4}{t+1} - 2t$, where time is in seconds and displacement in metres.

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| | Question | 27 | (6 | marks) |
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Marks

The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 1 \le x \le 4\\ 0 & otherwise \end{cases}$$

Where k is a constant

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Question 27 continued on next page

| Show that the | ne median of X lie | es between x | = 3.2 and $x = 1$ | 3.3 | |
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| Que | mestion 28 (3 marks) | | | | | | | |
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| Cons | sider the geometric series $1 + \frac{4}{3}\sin^2 x + \frac{16}{9}\sin^4 x + \frac{64}{27}\sin^6 x +$ | | | | | | | |
| (a) | When the limiting sum exists, find an expression for its value. | 1 | | | | | | |
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| (b) | For what values of x in the interval $0 < x < \frac{\pi}{2}$ does the limiting sum of this series | 2 | | | | | | |
| | exist? | | | | | | | |
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| Ques | stion 29 (4 marks) | | | | | | | |
| (a) | Find $\int \sec^2(2x)\tan^4(2x)dx$ | 2 | | | | | | |
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| (b) | Find $\int \frac{5x^2}{x^3 + 1} dx$ | 2 | | | | | | |
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Question 30 (4 marks)

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Luke suspects that the rate at which he spends cash is affected by the amount of cash he withdrew at his previous visit to an ATM.

The table below shows the amount of cash withdrawn, x, from an ATM, and the time, y hours, until Luke's next withdrawal from an ATM.

| Withdrawal | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------|----|----|-----|-----|-----|-----|----|-----|-----|-----|
| x | 40 | 10 | 100 | 110 | 120 | 150 | 20 | 90 | 80 | 130 |
| у | 56 | 62 | 195 | 330 | 94 | 270 | 48 | 196 | 214 | 286 |

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| | ation coefficient between x and y for the withdrawals 1 to 10. e of the correlation. |
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The table below shows the future values of an annuity, for different rates of interest and for different numbers of compounding periods, where contributions of \$1 are made at the end of each compounding period.

Table of future value interest factors

| n | 1% | 2% | 3% | 4% | 5% | 6% |
|---|--------|--------|--------|--------|--------|--------|
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 2.0100 | 2.0200 | 2.0300 | 2.0400 | 2.0500 | 2.0600 |
| 3 | 3.0301 | 3.0604 | 3.0909 | 3.1216 | 3.1525 | 3.1836 |
| 4 | 4.0604 | 4.1216 | 4.1836 | 4.2465 | 4.3101 | 4.3746 |
| 5 | 5.1010 | 5.2040 | 5.3091 | 5.4163 | 5.5256 | 5.6371 |
| 6 | 6.1520 | 6.3081 | 6.4684 | 6.6330 | 6.8019 | 6.9753 |

| (a) | An annuity account is opened and contributions of \$1200 are made at the end of each half year for 3 years at an interest rate of 4% p.a. compounding half yearly. Calculate the final amount in the account immediately after the last contribution is made. | 2 |
|-----|---|---|
| (b) | Calculate the single lump sum amount that would need to be invested at the start to reach the same final amount at the end of the 3 years with the same interest rate of 4% compounding half yearly. | 2 |
| | | |

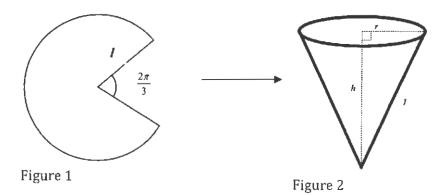
Question 32 (6 marks)

(b)

Marks

2

An open cone, of radius r cm and height h cm is made from a sector of a circle. The area of the sector used is 300 cm^2



Show from Figure 1 that the slant height l is given by $l^2 = \frac{450}{\pi}$

In Figure 2 it is given that $h=\sqrt{l^2-r^2}$ (do not prove this). Show that the volume of the cone is given by $V=\frac{1}{3}r^2\sqrt{450\pi-\pi^2r^2}$

Question 32 continued on next page

Question 32 continued

| Hence or otherw | vise, find the | value of r fo | or the volun | ne of the cone | to be a maxim | um. |
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End of paper

Section I

10 marks Attempt questions 1 - 10 Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. What is the value of $\csc \frac{\pi}{3}$ to three significant figures?

- (A) 1.00
- (B) 1.15
- (C) 1.41
- (D) 2.00

2. What is the value of *c* for which the circle $(x-3)^2 + (y-2)^2 = c$ touches the *x* axis?

- (A) 2
- (B) 3
- (C) 4
- (D) (

3. What is the equation of the tangent to $y = x^2 - 3$ at x = -1?

- (A) y = -2x 4
- (B) y = 2x 4
- (C) $y = \frac{x}{2} \frac{3}{2}$
- (D) $y = -\frac{x}{2} \frac{3}{2}$

- 4. Which statement is true for an ungrouped data set with no outliers?
 - (A) The largest possible range is 2 times the interquartile range.
 - (B) The largest possible range is 3 times the interquartile range.
 - The largest possible range is 4 times the interquartile range.
 - (D) The largest possible range is 5 times the interquartile range.
- 5. Which one of the following is the set of all solutions to $2x^2 5x + 2 \ge 0$?
 - (A) $\left[\frac{1}{2}, 2\right]$
 - (B) $\left(\frac{1}{2},2\right)$
 - (C) $\left(-\infty, \frac{1}{2}\right) \cup \left(2, \infty\right)$
- 6. The graph of y = f(x) has a stationary point at (2, -3).

Which one of the following is a guaranteed stationary point of $y = -f\left(\frac{x}{2}\right) - 5$?

- (A) (1,-2)
- (B) (1,2)
- (4,-2)
- (D) (4,2)

- 7. What is the period and amplitude for the curve $y = \sin \pi x$?
 - (A) Amplitude = 1; Period = 2
 - (B) Amplitude = π ; Period = 2
 - (C) Amplitude = 1; Period = 2π
 - (D) Amplitude = π ; Period = 2π
- 8. If the z scores on an examination are normally distributed and P(z < N) = 0.6 for some number N, what is the value of P(-N < z < N)?
 - (A) 0.1
 - (B) 0.2
 - (C) 0.3
 - (D) 0.4
- 9. Which one of the following equations is NOT correct?

(A)
$$\int x(x^2 - 1)^2 dx = \frac{(x^2 - 1)^3}{6} + c$$

(B)
$$\int_{-3}^{3} \sqrt{9 - x^2} \, dx = \frac{9\pi}{2}$$

(C)
$$\int_{-1}^{1} 3^{x} dx = \frac{1}{\ln 3} \left(3 - \frac{1}{3} \right)$$

$$\int_{-5}^{5} 4x^4 - x^3 + \cos x \, dx = 0$$

- 10. Consider the series $\sqrt{5} + \sqrt{45} + \sqrt{125} + \dots + z = 225\sqrt{5}$, the value of z is:
 - (A) $25\sqrt{5}$
 - (B) 29√5
 - (C) $30\sqrt{5}$
 - (D) $35\sqrt{5}$

Section II

90 marks

Attempt all questions

Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of the examination paper.

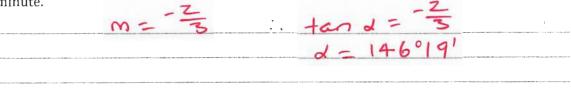
Question 11 (2 marks)

Marks

2

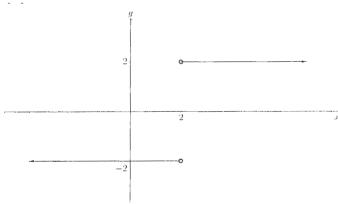
What angle does the line 2x + 3y + 6 = 0 make with the positive x-axis? Round to the nearest

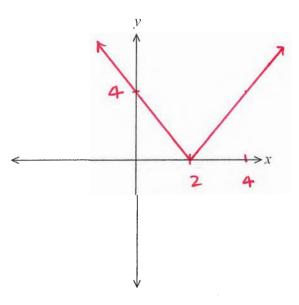
minute.



Question 12 (2 marks)

Sketch a possible function which could have the gradient function as graphed below.

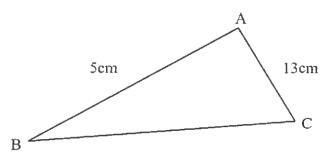




Question 13 (3 marks)

Marks

In triangle *ABC*, the length of *AB*=5cm, *AC*=13cm and $\cos \langle BAC = \frac{1}{8} \rangle$



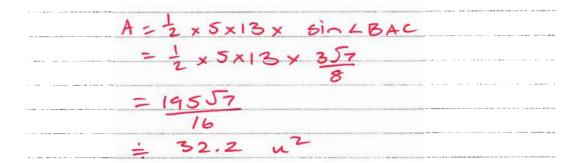
(a) Find the exact value of $sin\langle BAC \rangle$

2



(b) Find the area of triangle ABC

1



Question 14 (3 marks)

Solve $2 \log x = \log(5x + 6)$

$$\log x^{2} = \log (5x+6)$$

$$\therefore x^{2} = 5x+6$$

$$x^{2} - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6, -1$$

$$\therefore x = 6$$

Question 15 (3 marks)

Marks

Solve $\left|1-2\cos^2 x\right|=1$ for $0 \le x \le 2\pi$

3

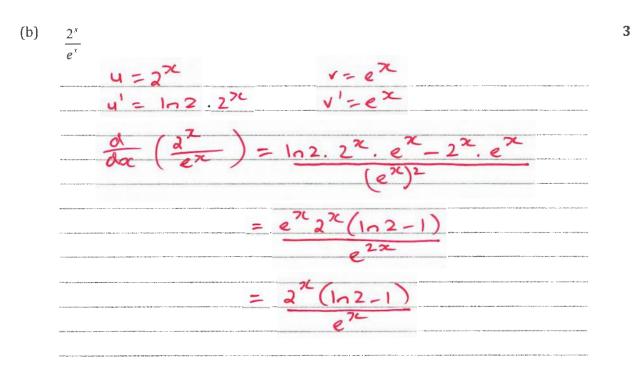
| 1-2 6052x=1 | 1-1-2652 x=-1 |
|--|------------------------|
| $2 \cos^2 x = 0$ | $2\cos^2x=2$ |
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Question 16 (5 marks)

Differentiate the following expressions.

Marks

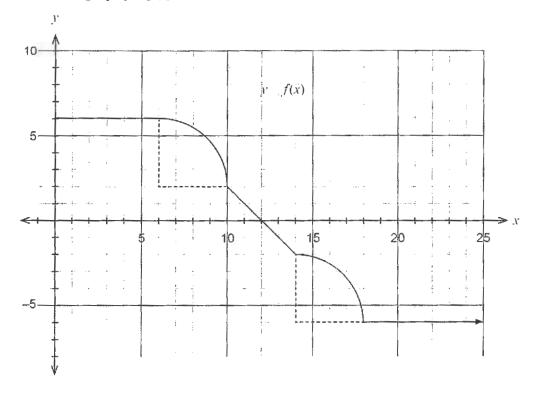
| (a) | $\frac{d}{dx}(\log_s(\tan x)) = \sec^2 x$ |
|-----|---|
| | n5 tanoc |
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Question 17 (5 marks)

Marks

Consider the graph y = f(x). Both arcs have a radius of four units.



Using the graph of y = f(x), $x \ge 0$, evaluate exactly the following integrals.

| $\int_{0}^{\infty} f(x) dx$ | | | | | | |
|--|---|--|---|--|--|--|
| = | (5x6) | + (6+8 |)+ 4x7 | Tx42+ | 1 × 2× | 2 |
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| (b) | $\int_{0}^{18} f(x)dx \qquad 12 \qquad \qquad 18$ | |
|-----|--|--|
| | = If(x) da + If(x) dsc | |
| | $= 46 + 4\pi - \left(\frac{1}{2} \times 2 \times 2 + \left(4 \times 6 - \frac{1}{4} \times \pi \times 4^{2}\right)\right)$ | |
| | = 46+4TT-(26-4TT) | |
| | = 20 +8# | |

Question 18 (5 marks)

Marks

The discrete random variable X has probability distribution shown in the table below

| x | -1 | 0 | 1 | 2 | 3 |
|--------|----|---|-----|------|------|
| P(X=x) | | b | 0.2 | 0.15 | 0.13 |

and E(X) = 0.55

| (a) By forming a pair of simultaneous equations, or otherwise, find the values of a and | b. 3 |
|---|------|
|---|------|

 $E(x) = \sum xi pi$ = -1(a) + 0(b) + 1(0.2) + 2(0.15) + 3(0.13) $\vdots \quad 0.55 = -a + 0.89$ a = 0.34

(b) Calculate
$$Var(X)$$

2

 $Var(X) = E(X^2) - \mu^2$

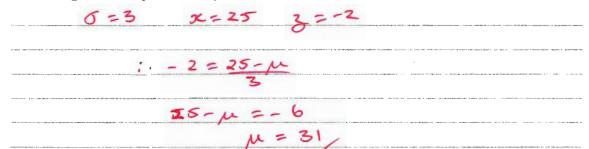
 $E(x^2) = 1(a) + 0(b) + 1(0.2) + 4(0.15) + 9(0.13)$ = 2.31

Question 19 (2 marks)

Marks

The length of steel rods produced by a machine is normally distributed with a standard deviation of 3 mm. It is found that 2.5% of all rods are less than 25 mm long. Find the mean length of rods produced by the machine.

2



Question 20 (8 marks)

Consider the function $f(x) = x^3 + 6x^2 + 9x + 4$ in the domain $-4 \le x \le 1$

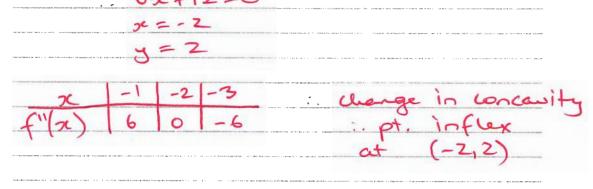
(a) Find the coordinates of any stationary points and determine their nature.

3

| $f'(x) = 3x^2 + 12x + 9$ $f''(x) = 6$ stat. pts at $f'(x) = 0$ | |
|---|------------|
| | |
| $3(x^2+4x+3)=0$ | |
| (26+3)(x+1)=0 | Page and a |
| | |
| y = 4 $y = 0$ | |
| f''(-3) = -18+1220 $f''(-1) = -1$ | 6+1270 |
| 1): max terning V: min to | vn. pt |
| pt at at | |
| (-3,4) $(-1,0)$ | |

(b) Determine the coordinates of its point(s) of inflexion.

2



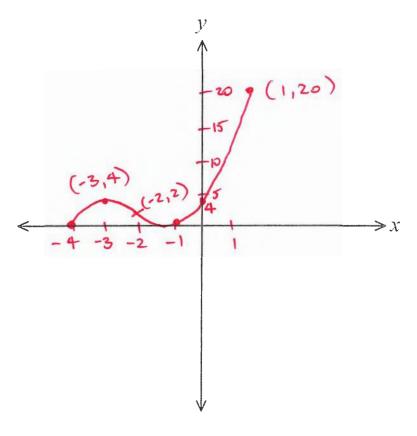
Question 20 continued on next page

Question 20 continued

Marks

(c) Draw a sketch of the curve y = f(x) in the domain $-4 \le x \le 1$ clearly showing all essential features.

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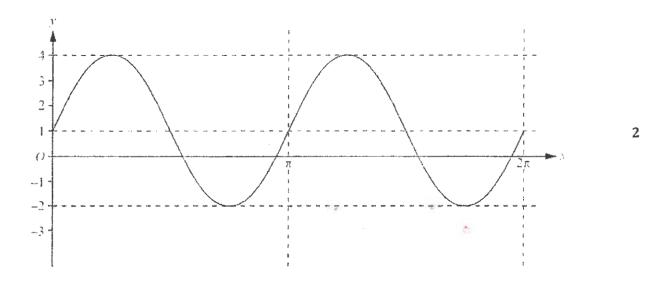


endpoints (-4,0) & (1,20) y-int = 4

(d) What is the global maximum value of the curve in the domain $-4 \le x \le 1$

Question 21 (2 marks)

The diagram shows the graph of $y = a \sin(bx) + c$ for $0 \le x \le 2\pi$, where a, b and c are positive integers.



Find the values of a, b and c.

| a = 3 | T=f | |
|-------|--|----------------------------|
| C=1 | :. 21 - 11 | |
| | 10 | |
| | :. 6=2 | |
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Question 22 (5 marks)

Marks

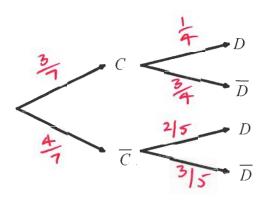
A pet ownership survey resulted in the following results:

$$P(C) = \frac{3}{7}$$
, $P(D \mid \overline{C}) = \frac{2}{5}$, and $P(\overline{D} \mid C) = \frac{3}{4}$.

Where C is the event that "a person has a cat" and D is the event that "a person has a dog"

(a) Complete the probability tree by marking a probability on each branch.

2



(b) If one person is chosen at random, find the probability that the person has:

| i) a cat and a | ı dog | |
|----------------|-------|--|
| | | |

1

| P | (c | 0 | D |) = | 3 | x | 4 |
|------|----|---|---|------------------------|---|---|---|
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ii) at least one pet (cat or dog)
$$P(at least 1pet) = 1 - P(DC)$$

$$= 1 - \frac{4}{7} \times \frac{3}{5}$$

$$= \frac{23}{35}$$

Question 23 (5 marks)

Marks

The function f is defined by $f(x) = 2 + \sqrt{x-3}$ for $x \ge 3$.

The function g is defined by $g(x) = \frac{12}{x} + 2$ for x > 0

Write the domain and range of the function f using interval notation. (a)

3



Write an expression for the composite function h(x) = g(f(x)) and hence find a (b) value for g(f(12))

2

$$g(f(x)) = \frac{12}{2 + \sqrt{2 - 3}} + \frac{12}{2}$$

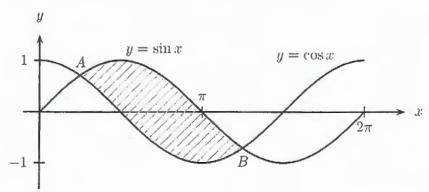
$$g(f(12)) = \frac{12}{2 + \sqrt{12 - 3}} + \frac{12}{2}$$

$$= \frac{22}{3}$$

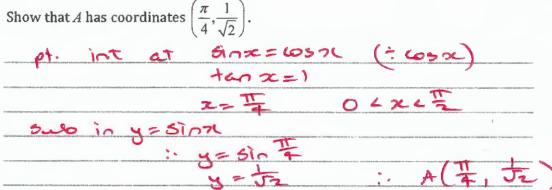
Question 24 (5 marks)

(a)

The diagram shows the graphs $y = \sin x$ and $y = \cos x$, $0 \le x \le 2\pi$. The graphs intersect at A and B.



2



Question 24 continued on next page

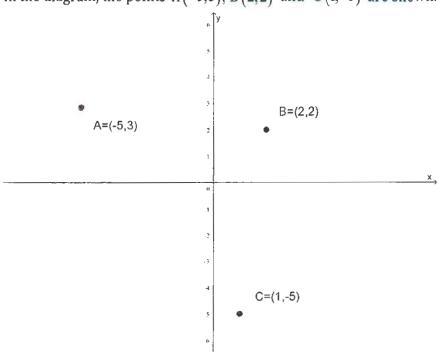
3

Question 24 continued

(b) Find the area enclosed by the two graphs.

| Solu | 2 for B. $\tan x = 1$ $\pi < x < \frac{3\pi}{2}$ |
|--|--|
| | : x = 5 = |
| | 5T 4 |
| A = | (sinx-ws re) dre |
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In the diagram, the points A(-5,3), B(2,2) and C(1,-5) are shown.



(a) Calculate the gradient of AC.

A applicated to an in-

(b) Find the coordinates of *X*, the midpoint of *AC*.

1

1

 $M = \begin{pmatrix} -541 \\ 2 \end{pmatrix}, \frac{3-5}{2} \end{pmatrix}$ = (-2, -1)

(c) Find the coordinates of *D* if *X* is also the midpoint of *BD*.

 $-2 = \frac{2+x}{2}$ $-1 = \frac{2+y}{2}$ -4 = 2+x 2+y = -2 y = -4

:. D(-6,-4)

Question 26 (5 marks)

Marks

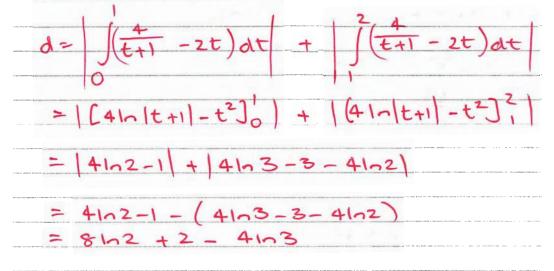
A particle is moving in a straight line. Its velocity for $t \ge 0$ is given by $v = \frac{4}{t+1} - 2t$, where time is in seconds and displacement in metres.

(a) Find when the particle changes direction.

2

| Change direction | at v=0 | |
|---------------------|------------|-----------|
| 4 -2t=0 | (t+2)(t-1) | =0 |
| t+1 | t=-2,1 | , t 70 |
| $4 - 2t^2 - 2t = 0$ | ;, t=1 | |
| t2+t-2=0 | | |

(b) Find the exact distance travelled in the first 2 seconds.



Question 27 (6 marks)

Marks

2

3

The continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 1 \le x \le 4 \\ 0 & otherwise \end{cases}$$

Where k is a constant

(a) Show that $k = \frac{1}{12}$

 $F(x) = k \int (x^2 - 2x + 2) dx$ $\therefore k \left(\frac{x}{5} - x^2 + 2x \right)^{\frac{1}{2}} = 1$

: k ((2 - 16:+8) - (3 - 1 + 2)) = 1

:. 12k=1 , k= 12/

(b) Fully define the cumulative distribution function F(x).

For 15x 4

 $=\frac{1}{12}\left[\frac{\chi^3}{5}-\chi^2+2\chi\right]^{\chi}$

 $=\frac{1}{12}\left[\frac{x^3}{3}-x^2+2x-\left(\frac{1}{3}-1+2\right)\right]$

=九【号- x2+2x-等】

= 36 - 2 + 5 - 5

 $F(x) = \begin{cases} 0 & x < 1 \\ \frac{23}{36} - \frac{x^2}{12} + \frac{x}{6} - \frac{1}{4} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$

Question 27 continued on next page

1

Question 27 continued

(c) Show that the median of X lies between x = 3.2 and x = 3.3

 $F(3.2) = \frac{3.2^{3}}{36} - \frac{3.2^{2}}{12} + \frac{3.2}{6} - \frac{1}{9}$ $= 0.479 \le 0.5$ $F(3.3) = \frac{3.3^{3}}{36} - \frac{3.3^{2}}{12} + \frac{3.3}{6} - \frac{1}{9}$ = 0.5296 > 0.5

F(median) = 0.5 : median lies between x = 3.2

Question 28 (3 marks)

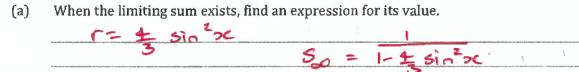
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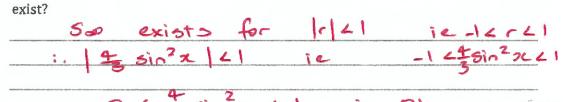
2

Consider the geometric series $1 + \frac{4}{3}\sin^2 x + \frac{16}{9}\sin^4 x + \frac{64}{27}\sin^6 x + ...$



a=1

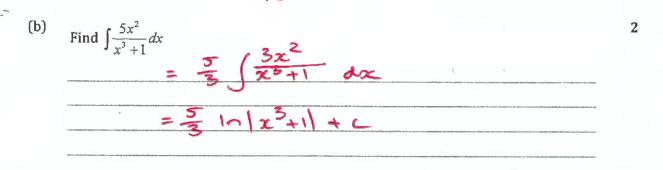
(b) For what values of x in the interval $0 < x < \frac{\pi}{2}$ does the limiting sum of this series



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Question 29 (4 marks)

(a) Find $\int \sec^2(2x) \tan^4(2x) dx$ = $\frac{1}{2} \int 2 \sec^2(2x) \tan^4(2x) dx$ = $\frac{1}{2} \tan^5(2x) + C$



Question 30 (4 marks)

Marks

2

Luke suspects that the rate at which he spends cash is affected by the amount of cash he withdrew at his previous visit to an ATM.

The table below shows the amount of cash withdrawn, x, from an ATM, and the time, y hours, until Luke's next withdrawal from an ATM.

| Withdrawal | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------|----|----|-----|-----|-----|-----|----|-----|-----|-----|
| x | 40 | 10 | 100 | 110 | 120 | 150 | 20 | 90 | 80 | 130 |
| у | 56 | 62 | 195 | 330 | 94 | 270 | 48 | 196 | 214 | 286 |

| (a) | Find the equation of the least squares regression line for y in terms of x , for the |
|-----|--|
| | withdrawals 1 to 10 and hence estimate how much cash (to the nearest \$10) Luke |
| | would need to withdraw from the ATM at his previous visit in order to not need to |
| | visit an ATM again for 120 hours. |
| | |

visit an ATM again for 120 hours.

from calculator: y = A + Bx A = 30.26 y = 1.7x + 30.26

Find x at y = 120 120 = 1.7x + 30.26 x = 52.79x = 52.79

| (b) | Calculate the correlation coefficient between x and y for the withdrawals 1 to | 10. |
|-----|--|-----|
| | Describe the nature of the correlation | |

The table below shows the future values of an annuity, for different rates of interest and for different numbers of compounding periods, where contributions of \$1 are made at the end of each compounding period.

Table of future value interest factors

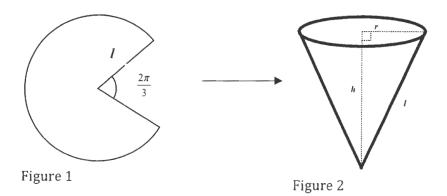
| | javar o ranao | | | | | |
|---|---------------|--------|--------|--------|--------|--------|
| n | 1% | 2% | 3% | 4% | 5% | 6% |
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 2.0100 | 2.0200 | 2.0300 | 2.0400 | 2.0500 | 2.0600 |
| 3 | 3.0301 | 3.0604 | 3.0909 | 3.1216 | 3.1525 | 3.1836 |
| 4 | 4.0604 | 4.1216 | 4.1836 | 4.2465 | 4.3101 | 4.3746 |
| 5 | 5.1010 | 5.2040 | 5.3091 | 5.4163 | 5.5256 | 5.6371 |
| 6 | 6.1520 | 6.3081 | 6.4684 | 6.6330 | 6.8019 | 6.9753 |

| made | late the final amount in the account immediately $FV = 1700 \times 6.3081$ | r= 2% half | u |
|--------|--|--------------|---|
| | = \$7569.72 | n= 6 | 3 |
| to rea | late the single lump sum amount that would need to the same final amount at the end of the 3 year of 4% compounding half yearly. | | 2 |
| | | | |

Question 32 (6 marks)

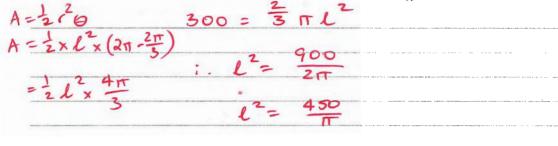
Marks

An open cone, of radius r cm and height h cm is made from a sector of a circle. The area of the sector used is $300~\rm cm^2$



(a) Show from Figure 1 that the slant height *l* is given by $l^2 = \frac{450}{\pi}$





(b) In Figure 2 it is given that $h = \sqrt{l^2 - r^2}$ (do not prove this).

2

Show that the volume of the cone is given by $V = \frac{1}{3}r^2\sqrt{450\pi - \pi^2r^2}$

V= 3πr2/12-r2

 $=\frac{1}{3}\pi r^2 \sqrt{\frac{450}{11}} - r^2$

 $=\frac{1}{3}r^2\sqrt{450\pi-\pi^2r^2}$

Question 32 continued on next page

2

Question 32 continued

(c) It is known that $\frac{dV}{dr} = \frac{300\pi r - \pi^2 r^3}{\sqrt{450\pi - \pi^2 r^2}}$ (do not prove this).

Hence or otherwise, find the value of r for the volume of the cone to be a maximum.

stat et at T = 0

: 300 Tr - T2 (3 = 0

 $:: \pi r (300 - \pi r^2) = 0 \qquad r \neq 0$

T (2 = 300

r = \\ \frac{300}{17} = \land{10.53}

r= /300 V m

End of paper