Student name: ________________

2020

YEAR 12
YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt all questions
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Simplify $\dfrac{x^2+5x+6}{x^2-9}$

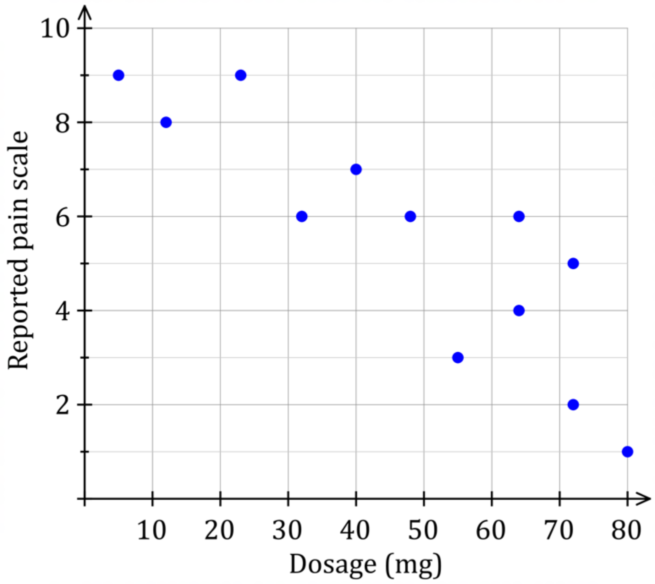
   (A) $\dfrac{x+2}{x-3}$

   (B) $\dfrac{x+3}{x-3}$

   (C) $\dfrac{x+2}{x+3}$

   (D) $\dfrac{x-2}{x-3}$

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation

(B) A moderate positive correlation

(C) A weak positive correlation.

(D) No correlation.

3. What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

4. What is the period and amplitude for the curve $y = \sin \pi x$?

(A) Amplitude = 1; Period = 2

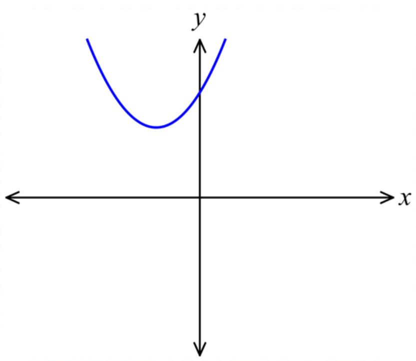
(B) Amplitude = π; Period = 2

(C) Amplitude = 1; Period = 2π

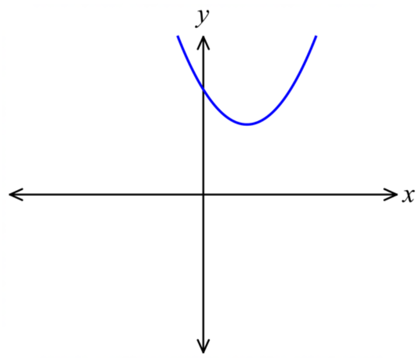
(D) Amplitude = π; Period = 2π

5. Which diagram best shows the graph of the parabola $y = 2 - (x+1)^2$

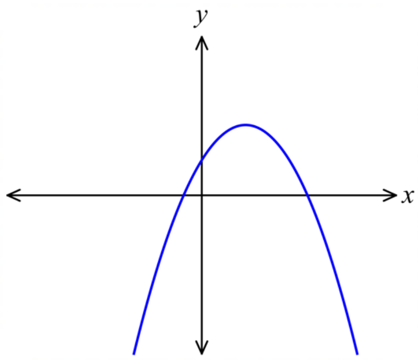
(A)



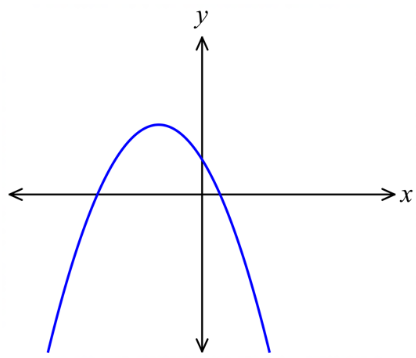
(B)



(C)



(D)



6. The equation of the least-squares regression line is given by $y = mx + c$, where $\boldsymbol{m} = \boldsymbol{r}\,\frac{s_y}{s_x}$ with r=correlation coefficient and s_x and s_y are the sample standard deviations.
What is the slope (m) of the least-squares regression line $y = mx + c$, if $r = 0.675$, $s_x = 2.567$ and $s_y = 4.983$?

(A) 0.35

(B) 1.31

(C) 1.70

(D) 3.36

7. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

(A) $3x^3(4-x)^3(7x-16)$

(B) $3x^3(4-x)^3(16-7x)$

(C) $3x^3(4-x)^2(7x-16)$

(D) $3x^3(4-x)^2(16-7x)$

8. The mean of a set of data is 14 and the standard deviation is 2.1.
If each score in the data set is increased by 4, which of the following statements will be true?

(A) The mean and standard deviation will increase by 4

(B) The mean will increase by 4 and the standard deviation will not change

(C) The mean will not change, and the standard deviation will increase by 4

(D) The mean and standard deviation will increase by a factor of 4

9. What are the solutions to the equation $2\sin x + \sqrt{3} = 0$, where $\{x: 0 \leq x \leq 2\pi\}$?

(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

(B) $\frac{2\pi}{3}, \frac{5\pi}{3}$

(C) $\frac{4\pi}{3}, \frac{5\pi}{3}$

(D) $\frac{7\pi}{3}, \frac{11\pi}{3}$

10. The table below shows the future value of a $1 annuity.

Future value of $1				
End of year	4%	6%	8%	10%
1	1.00	1.00	1.00	1.00
2	2.04	2.06	2.08	2.10
3	3.12	3.18	3.25	3.31
4	4.25	4.37	4.51	4.64

What amount would need to be invested every month into an account earning 16% p.a. interest compounded quarterly, to be worth $28 475 after a year?

(A) $6137

(B) $6314

(C) $6700

(D) $13 958

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

Question 11 (2 marks) **Marks**

Find the anti-derivative of $\dfrac{1}{1-2x}$ with respect to *x*. **2**

Question 12 (3 marks)

Let $f(x) = \dfrac{(x+3)(2x+1)}{\sqrt{x}}, x > 0$

(a) Show that $f(x)$ can be written in the form **2**

$Ax^{\frac{3}{2}} + Bx^{\frac{1}{2}} + Cx^{-\frac{1}{2}}$

Find the values of A, B and C.

(b) Find $f'(x)$ **1**

Question 13 (3 marks) **Marks**

The random variable X has this probability distribution.

X	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$ **1**

(b) Find the variance of X. **2**

Question 14 (2 marks)

Find $\int 6x^2 + 2 + x^{-\frac{1}{2}}\,dx$, giving each term in its simplest form. **2**

Question 15 (2 marks)

Find the common ratio of a geometric series with a first term of 3 and a limiting sum of 1.8. **2**

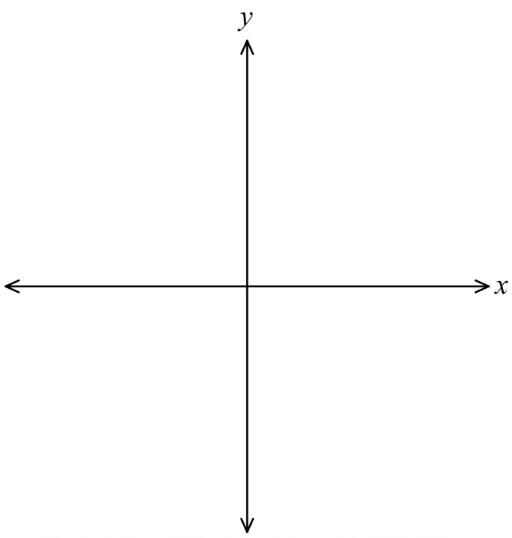
Question 16 (8 marks) **Marks**

Let $f(x) = (x - 2)(x^2 + 1)$

(a) Find where the graph of $y = f(x)$ cuts the x-axis and y axis. **2**

(b) Find the coordinates of the stationary points on the curve with the equation $y = f(x)$ and determine their nature. **3**

(c) Sketch the graphs of $\boldsymbol{y = f(x)}$ and $\boldsymbol{y = -f(x)}$ on the same diagram. **3**



Question 17 (2 marks)

Find the exact value of $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$. **2**

Question 18 (5 marks) **Marks**

Given that $f(x) = (x^2 - 6x)(x - 3) + 2x$.

(a) Express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants. **2**

(b) Hence factorise $f(x)$ completely. **1**

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. **2**

Question 19 (3 marks)

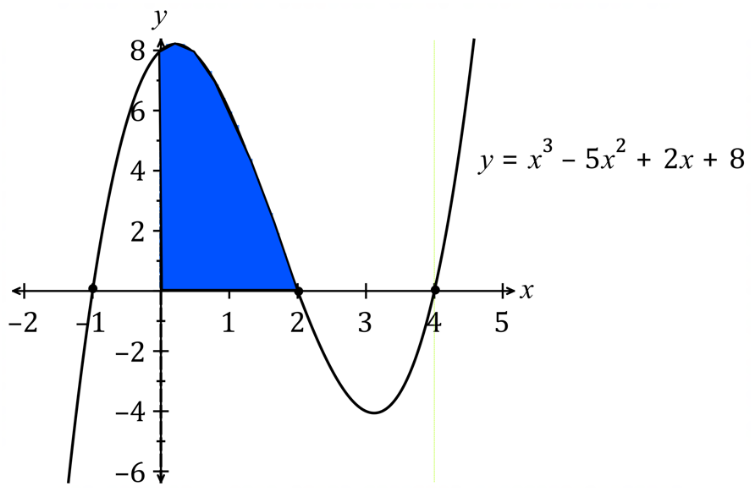
Differentiate with respect to x

(a) $\ln(x^2 + 2)$ **1**

(b) $\dfrac{\sin x}{x^2}$ **2**

Question 20 (2 marks)

Marks



Find the shaded area enclosed by the curve $y = x^3 - 5x^2 + 2x + 8$ and the coordinate axes. **2**

Question 21 (4 marks)

(a) Evaluate $\sum_{k=1}^{4}(-1)^k k^2$ **2**

(b) A tree grows from ground level to a height of 1.2 metres in one year. In each subsequent year, it grows $\frac{9}{10}$ as much as it did in the previous year. **2**

Find the limiting height of the tree.

Question 22 (6 marks) **Marks**

Sonny repays a loan over a period of n months. His monthly repayments form an arithmetic sequence. He repays \$119 in the first month, \$117 in the second month, \$115 in the third month, and so on. Sonny makes his final repayment in the nth month.

(a) Find the amount Sonny repays in the 25th month. **2**

(b) Over the n months, he repays a total of \$3200. **2**
Form an equation in n, and show that your equation may be written as
$n^2 - 120n + 3200 = 0$.

(c) State, with a reason, which of the solutions to the equation in part (b) is not a sensible solution to the repayment problem. **2**

Question 23 (2 marks)

If $\tan\theta = \frac{2}{3}$, and θ is acute, find the exact value of $\sin\theta$? **2**

Question 24 (6 marks) **Marks**

A curve with the equation $y = f(x)$, has $\dfrac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\dfrac{d^2y}{dx^2}$ **1**

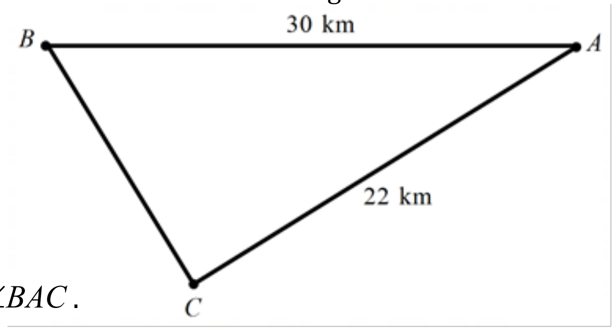
(b) Show that $\dfrac{d^2y}{dx^2} \geq 2$ for all values of x. **1**

(c) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

(d) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

Question 25 (3 marks) **Marks**

Towns A, B and C are to be connected by high-speed optic fibre cables.
Town B is due west of town A. Town C is on a bearing of $210°$ from Town A.



NOT TO SCALE

(a) Find the size of $\angle BAC$. **1**

....

....

....

....

(b) Find the distance, to the nearest kilometre, between towns B and C. **2**

....

....

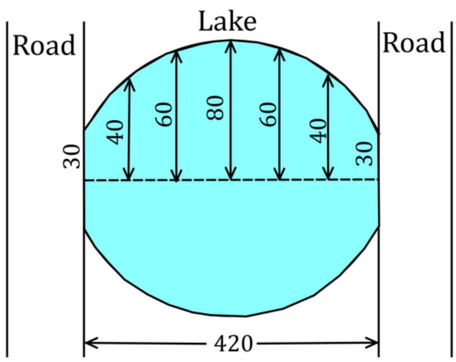
....

....

....

Question 26 (3 marks)

A symmetrical lake has two roads, 420 metres apart, forming two of its sides.



Not to scale

Equally spaced measurements of the lake, in metres, are shown on the above diagram. Use the trapezoidal rule to estimate the area of the lake. **3**

....

....

....

....

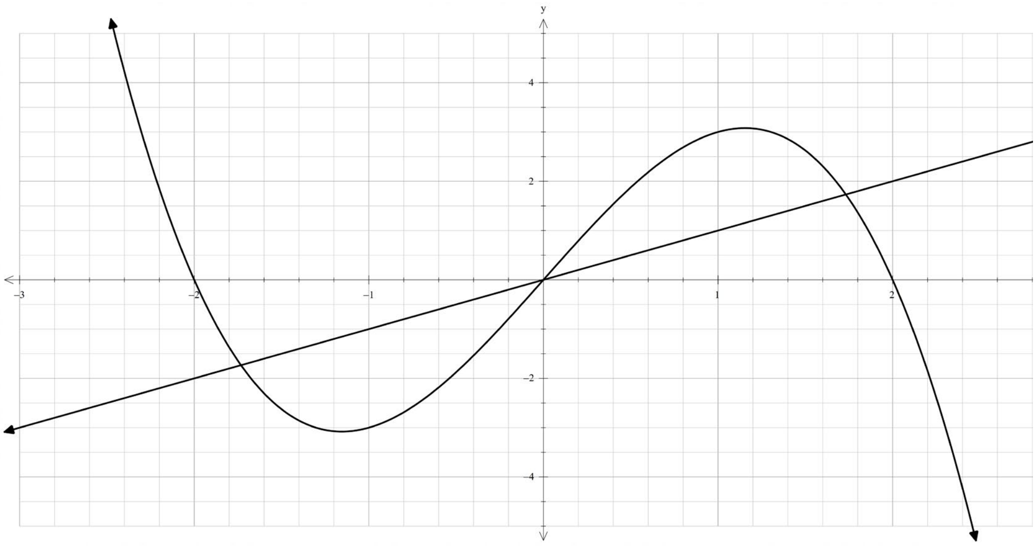
....

....

....

Question 27 (4 marks) **Marks**

The functions $y = -x^3 + 4x$ and $y = x$ are sketched below.



(a) Show that the functions intersect when $x = 0$ and $x = \pm\sqrt{3}$. **2**

(b) Hence find the exact area between the two functions in the first quadrant. **2**

Question 28 (3 marks)

The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population P of a nest of this insect grows exponentially so that: **3**

$$\frac{dP}{dt} = 1200e^{0\cdot 3t}$$

A nest of these insects had a population of 5000 after one month.

Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Answer correct to the nearest month.

Question 29 (3 marks)

A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.

(a) Draw a tree diagram to show the possible outcomes. Include the probability on each branch. **2**

(b) What is the probability that the two lollies are of different colours? **1**

Question 30 (7 marks) **Marks**

A particle moves along a straight line so that its displacement, x metres, from a fixed point O is given by $x = 1 + 3cos2t$, where t is measured in seconds.

(a) What is the initial displacement of the particle? **1**

(b) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

(c) Hence, or otherwise, find when AND where the particle first comes to rest after $t = 0$. **2**

(d) Find a time when the particle reaches its greatest magnitude of velocity. What is this velocity? **2**

Question 31 (2 marks) **Marks**

Find all solutions of $2sin^2x + cosx - 2 = 0$, where $0 \le x \le 2\pi$. **2**

Question 32 (3 marks)

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

(a) Find the present value, if $3200 is contributed per month for 46 months at 0.75% per month. Answer to the nearest cent. **1**

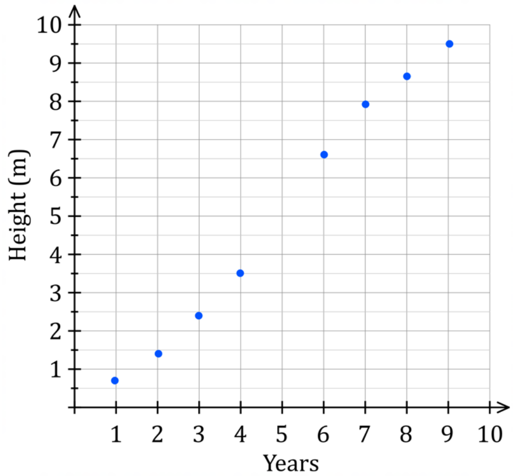
(b) Annabelle borrows $27 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 7.8% per annum interest. Find Annabelle's monthly repayment. Answer to the nearest cent. **2**

Question 33 (6 marks) **Marks**

Hayden is an agricultural scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5		6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. **1**

(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. **2**

(c) Hayden did not record the tree's height after five years. Predict the height after five years, correct to one decimal place. **1**

(d) Use algebra to estimate how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

(e) Comment on the reliability of your answers in (c) and (d). **1**

Question 34 (6 marks) **Marks**

The height $h(t)$ metres of the tide above the mean sea level on 1st April is given by the following rule:

$$h(t) = 4\sin\left(\frac{\pi}{8}t\right)$$

where t is the number of hours after midnight

(a) Draw a graph of $y = h(t)$ for $0 \leq t \leq 24$. **2**

(b) When was high tide? **1**

(c) What was the height of the high tide? **1**

(d) What was the height of the tide at 10 a.m.? **2**
Answer correct to one decimal place

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

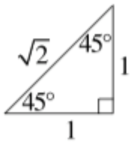
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

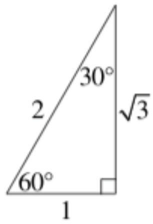
$A = \frac{1}{2}ab\sin C$



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \frac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

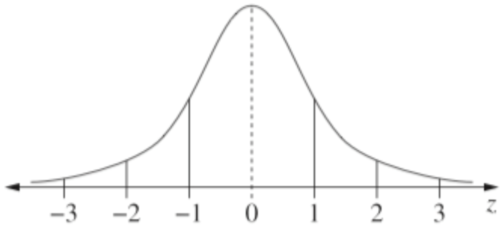
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Student name:

2020	**YEAR 12** YEARLY EXAMINATION

Mathematics Advanced

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt all questions
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Simplify $\dfrac{x^2+5x+6}{x^2-9}$

$\dfrac{x^2+5x+6}{x^2-9} = \dfrac{(x+3)(x+2)}{(x+3)(x-3)}$

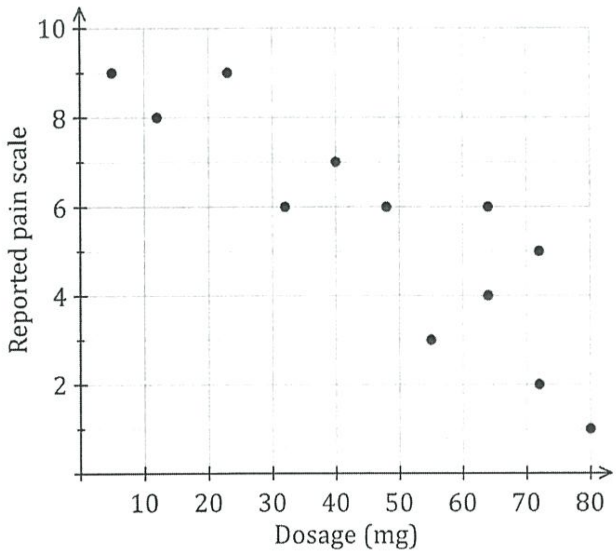
(A) $\dfrac{x+2}{x-3}$ ✓

(B) $\dfrac{x+3}{x-3}$

(C) $\dfrac{x+2}{x+3}$

(D) $\dfrac{x-2}{x-3}$

2. A scatterplot of pain (as reported by patients) compared to the dosage (in mg) of a drug is shown below.



How could you describe the correlation between the pain and the dosage?

(A) A moderate negative correlation ✓

(B) A moderate positive correlation

(C) A weak positive correlation.

(D) No correlation.

3. What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

$f'(x) = 6x^2 + 2x$
$f''(x) = 12x + 2$
Concave down $\cap$: $f''(x) < 0$
then $12x + 2 < 0$ | -2
$12x < -2$ | $\div 12$
$x < -\frac{2}{12}$
$x < -\frac{1}{6}$ ✓

4. What is the period and amplitude for the curve $y = \sin \pi x$?

(A) Amplitude = 1; Period = 2

(B) Amplitude = π; Period = 2

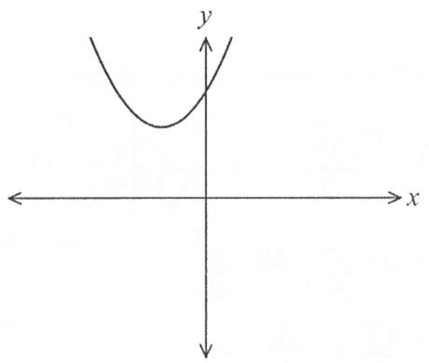
(C) Amplitude = 1; Period = 2π

(D) Amplitude = π; Period = 2π

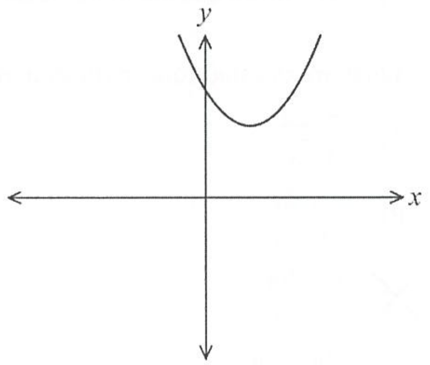
Amplitude = 1
period = $\frac{2\pi}{n}$ with $n = \pi$
$\therefore \frac{2\pi}{\pi} = 2$ ✓

5. Which diagram best shows the graph of the parabola $y = 2 - (x + 1)^2$

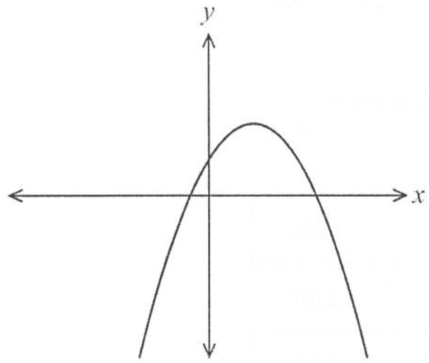
(A)



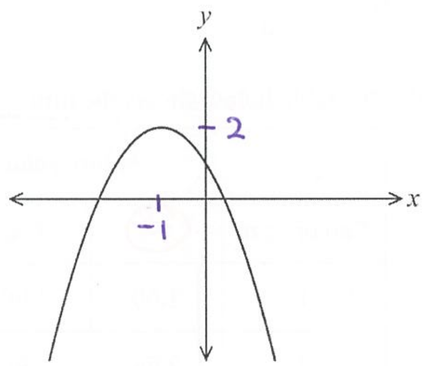
(B)



(C)



(D)



✓

✓

6. What is the slope of the least squares regression line $y = mx + c$, if $r = 0.675$, $s_x = 2.567$ and $s_y = 4.983$?

(A) 0.35

(B) 1.31

(C) 1.70

(D) 3.36

using $m = r\frac{S_y}{S_x}$

7. What is the value of $f'(x)$ if $f(x) = 3x^4(4-x)^3$?

(A) $3x^3(4-x)^3(7x-16)$

(B) $3x^3(4-x)^3(16-7x)$

(C) $3x^3(4-x)^2(7x-16)$

~~(D)~~ $3x^3(4-x)^2(16-7x)$

product rule

$f'(x) = 12x^3(4-x)^3 + 3x^4 \times 3(4-x)^2 \times (-1)$
$= 12x^3(4-x)^3 - 9x^4(4-x)^2$
$= 3x^3(4-x)^2(4(4-x) - 3x)$
$= 3x^3(4-x)^2(16-7x)$ ✓

✓ 8. The mean of a set of data is 14 and the standard deviation is 2.1.
If each score in the data set is increased by 4, which of the following statements will be true?

(A) The mean and standard deviation will increase by 4

~~(B)~~ The mean will increase by 4 and the standard deviation will not change

(C) The mean will not change, and the standard deviation will increase by 4

(D) The mean and standard deviation will increase by a factor of 4

✓ 9. What are the solutions to the equation $2\sin x + \sqrt{3} = 0$, where $\{x: 0 \le x \le 2\pi\}$?

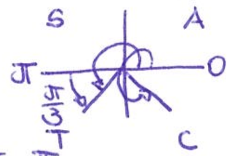
(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

(B) $\frac{2\pi}{3}, \frac{5\pi}{3}$

~~(C)~~ $\frac{4\pi}{3}, \frac{5\pi}{3}$

(D) $\frac{7\pi}{3}, \frac{11\pi}{3}$

$\sin x = -\frac{\sqrt{3}}{2}$



$\alpha = \frac{\pi}{3}$ (60°)

$x = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$
$= \frac{4\pi}{3}, \frac{5\pi}{3}$ ✓

10. The table below shows the future value of a \$1 annuity.

Future value of \$1				
End of year	4%	6%	8%	10%
1	1.00	1.00	1.00	1.00
2	2.04	2.06	2.08	2.10
3	3.12	3.18	3.25	3.31
4	4.25	4.37	4.51	4.64

What amount would need to be invested every month into an account earning 16% p.a. interest compounded quarterly, to be worth \$28 475 after a year?

(A) \$6137

(B) \$6314

~~(C)~~ \$6700

(D) \$13 958

4 quarters / year
at 16% ÷ 4 = 4% per quarter

$FV = 4.25 \times x$
$28475 = 4.25 \times x$
$x = \frac{28475}{4.25} = 6700$ ✓

Section II

90 marks
Attempt all questions
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

✓ **Question 11** (2 marks) **Marks**

Find the anti-derivative of $\frac{1}{1-2x}$ with respect to x. **2**

$-\frac{1}{2}\int \frac{1\times(-2)}{(1-2x)} = -\frac{1}{2}\ln|1-2x| + C$

$f(x) = -2x$
$f'(x) = -2$

Question 12 (3 marks)

Let $f(x) = \frac{(x+3)(2x+1)}{\sqrt{x}}, x > 0$

(a) Show that $f(x)$ can be written in the form **2**

$Ax^{\frac{3}{2}} + Bx^{\frac{1}{2}} + Cx^{-\frac{1}{2}}$

Find the values of A, B and C.

using: $\sqrt{x} = x^{\frac{1}{2}}$

$f(x) = \frac{2x^2+7x+3}{\sqrt{x}}$

$= \frac{2x^2}{\sqrt{x}} + \frac{7x}{\sqrt{x}} + \frac{3}{\sqrt{x}}$

$= 2x^2x^{-\frac{1}{2}} + 7x^1x^{-\frac{1}{2}} + 3x^{-\frac{1}{2}}$

$= 2x^{\frac{3}{2}} + 7x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ $\therefore A=2, B=7, C=3$

(b) Find $f'(x)$ **1**

$f(x) = 2x^{\frac{3}{2}} + 7x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$

$f'(x) = 2\times\frac{3}{2}x^{\frac{1}{2}} + 7\times\frac{1}{2}x^{-\frac{1}{2}} + 3\times(-\frac{1}{2})x^{-\frac{3}{2}}$

$= 3x^{\frac{1}{2}} + \frac{7}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$

Question 13 (3 marks) **Marks**

The random variable X has this probability distribution.

X	0	1	2	3	4
$P(X=x)$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$ **1**

$= 0.4 + 0.2$

$= 0.6$

(b) Find the variance of X. **2**

$\mu = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$

$= 2$

$Var(X) = E(X^2) - \mu^2$

$= 0^2 \times 0.1 + 1^2 \times 0.2 + 2^2 \times 0.4 + 3^2 \times 0.2 + 4^2 \times 0.1 - 2^2$

$= 1.2$

$\therefore$ Variance is 1.2

Question 14 (2 marks)

Find $\int 6x^2 + 2 + x^{-\frac{1}{2}}\,dx$, giving each term in its simplest form. **2**

$= \frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 2x^3 + 2x + 2x^{\frac{1}{2}} + C$

Question 15 (2 marks)

Find the common ratio of a geometric series with a first term of 3 and a limiting sum of 1.8. **2**

$S_\infty = \frac{a}{1-r}$

Sub: $1.8 = \frac{3}{(1-r)}$

$1 - r = \frac{3}{1.8}$ $\quad -1$

$-r = \frac{5}{3} - 1$

$-r = \frac{2}{3}$

$\therefore r = -\frac{2}{3}$

or $1.8(1-r) = 3$

$1.8 - 1.8r = 3$

$-1.8r = 3 - 1.8$

$-1.8r = 1.2$

$r = -\frac{1.2}{1.8} = -\frac{2}{3}$

Question 16 (8 marks) **Marks**

Let $f(x) = (x-2)(x^2+1)$

(a) Find where the graph of $y = f(x)$ cuts the x-axis and y axis. **2**

x-intercept: $y=0$
$0=(x-2)(x^2+1)$
$x=2$ $\quad x^2+1\neq 0$

y-intercept: $x=0$
$y=(0-2)(0^2+1)$
$=-2$

$\therefore$ x-intercept is 2
y-intercept is −2

(b) Find the coordinates of the stationary points on the curve with the equation $y = f(x)$ and determine their nature. **3**

$f(x) = (x-2)(x^2+1)$
$= x^3 + x - 2x^2 - 2$
$= x^3 - 2x^2 + x - 2$
$f'(x) = 3x^2 - 4x + 1$
$f''(x) = 6x - 4$

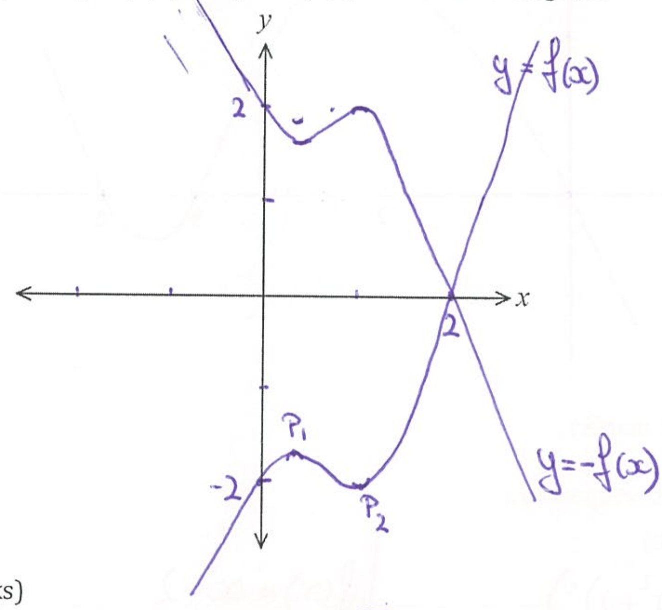
Stationary pts: $f'(x)=0$
$3x^2-4x+1=0$
$(3x-1)(x-1)=0$
$x=\frac{1}{3}$ or $x=1$

when $x=\frac{1}{3}$ then $y=-\frac{50}{27}$
when $x=1$ then $y=-2$
$\therefore P_1\left(\frac{1}{3}, -\frac{50}{27}\right)$ and $P_2(1,-2)$

P_1: $f''(x)=6\times\frac{1}{3}-4=-2<0 \therefore$ Max
P_2: $f''(x)=6\times 1-4=2>0 \therefore$ Min

$3x^2-4x+1 = (3x-1)(x-1)$

(c) Sketch the graphs of $y = f(x)$ and $y = -f(x)$ on the same diagram. **3**



Question 17 (2 marks)

Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx.$ **2**

$= \left[\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right)$

$= 1 - \frac{1}{\sqrt{2}}$

$\left(= \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2} \right)$

Question 18 (5 marks) **Marks**

Given that $f(x) = (x^2 - 6x)(x - 3) + 2x$.

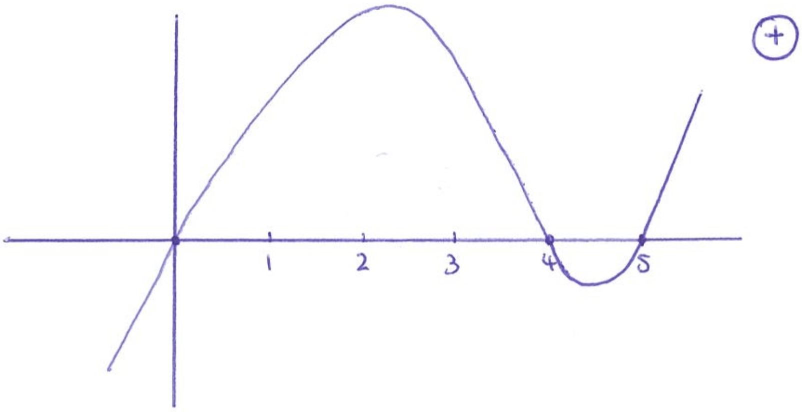
(a) Express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants. **2**

$f(x) = x^3 - 3x^2 - 6x^2 + 18x + 2x$
$= x^3 - 9x^2 + 20x$
$= x(x^2 - 9x + 20)$

(b) Hence factorise $f(x)$ completely. **1**

$f(x) = x(x^2 - 9x + 20)$
$= x(x-5)(x-4)$

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. **2**



√ **Question 19** (3 marks)

Differentiate with respect to x

(a) $\ln(x^2 + 2)$ **1**

$y = \ln(x^2+2)$
$y' = \frac{2x}{x^2+2}$

$f(x) = x^2 + 2$
$f'(x) = 2x$

(b) $\frac{\sin x}{x^2}$ **2**

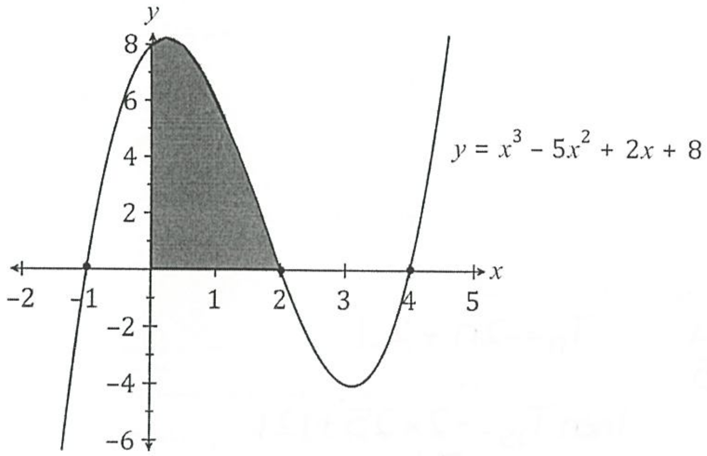
$y = \frac{\sin x}{x^2}$ $y' = \frac{x^2 \times \cos x - \sin(x) \times 2x}{(x^2)^2}$

$= \frac{x^2 \times \cos x - 2x \sin x}{x^4}$

$= \frac{x \cos x - 2 \sin x}{x^3}$

Question 20 (2 marks) **Marks**

2



Find the shaded area enclosed by the curve $y = x^3 - 5x^2 + 2x + 8$ and the coordinate axes.

$A = \int_0^2 (x^3 - 5x^2 + 2x + 8)\,dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{2x^2}{2} + 8x\right]_0^2$

$= \left(\frac{(2)^4}{4} - \frac{5(2)^3}{3} + (2)^2 + 8\times(2)\right) - 0 = \frac{32}{3} = 10\frac{2}{3}$ units^2

Question 21 (4 marks)

(a) Evaluate $\sum_{k=1}^{4}(-1)^k k^2$ 2

$= (-1)^1 \times 1^2 + (-1)^2 \times 2^2 + (-1)^3 \times 3^2 + (-1)^4 \times 4^2$

$= -1 + 4 - 9 + 16 = 10$

(b) A tree grows from ground level to a height of 1.2 metres in one year. In each subsequent year, it grows $\frac{9}{10}$ as much as it did in the previous year. 2

Find the limiting height of the tree.

$a = 1.2 \quad r = \frac{9}{10} \quad |r| < 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{1.2}{1-\left(\frac{9}{10}\right)} = 12\text{ m}$ (limiting height)

✓ **Question 22** (6 marks) **Marks**

Sonny repays a loan over a period of n months. His monthly repayments form an arithmetic sequence. He repays \$119 in the first month, \$117 in the second month, \$115 in the third month, and so on. Sonny makes his final repayment in the nth month.

(a) Find the amount Sonny repays in the 25th month. **2**

1st 2nd 3rd $T_n = -2n + 121$

119 $\underset{-2}{\curvearrowright}$ 117 115

then $T_{25} = -2 \times 25 + 121$

$= 71$

Repayment is \$71

(b) Over the n months, he repays a total of \$3200. **2**
Form an equation in n, and show that your equation may be written as
$n^2 - 120n + 3200 = 0$.

$S_n = 3200$ $a = 119$ $d = -2$

Sub.:

$S_n = \frac{n}{2}[2a + (n-1)d] \rightarrow 3200 = \frac{n}{2}[2 \times 119 + (n-1) \times (-2)]$

$6400 = n(238 - 2n + 2) = n(240 - 2n)$

$6400 = 240n - 2n^2 \Rightarrow n^2 - 120n + 3200 = 0$

(c) State, with a reason, which of the solutions to the equation in part (b) is not a sensible solution to the repayment problem. **2**

using $n^2 - 120n + 3200 = 0$

$(n-80)(n-40) = 0$

$n = 80$ or $n = 40$

when $n = 40$ then $T_{40} = -2 \times 40 + 121 = \$41$ ✓

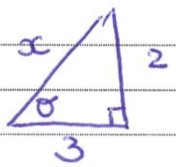
when $n = 80$ then $T_{80} = -2 \times 80 + 121 = \$-39$ ✗

↑ repayment is ⊖

∴ no sensible solution

✓ **Question 23** (2 marks)

If $\tan\theta = \frac{2}{3}$, and θ is acute, find the exact value of $\sin\theta$? **2**



$x = \sqrt{13}$

then $\sin\theta = \frac{2}{\sqrt{13}}$

Question 24 (6 marks) **Marks**

✓ A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(a) Find $\frac{d^2y}{dx^2}$ $= 3x^2 + 2x$ **1**

(b) Show that $\frac{d^2y}{dx^2} \geq 2$ for all values of x. **1**

$x^2 \geq 0$ then $3x^2 \geq 0$

$\therefore \frac{d^2y}{dx^2} \geq 2$

✓ (c) The point $P(2, 4)$ lies on the curve. Find y in terms of x. **2**

$y' = x^3 + 2x - 7$

$y = \frac{x^4}{4} + x^2 - 7x + C$

$P(2,4): \; 4 = \frac{(2)^4}{4} + (2)^2 - 7 \times (2) + C$

$\to C = 10$

$\therefore y = \frac{x^4}{4} + x^2 - 7x + 10$

✓ (d) Find an equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

$y' = x^3 + 2x - 7$

Gradient of the tangent at $P(2,4)$

$m = y' = 2^3 + 2 \times 2 - 7$

$m_1 = 5$

normal: $m_2 = -\frac{1}{m_1}$

$m_2 = -\frac{1}{5}$

$\to$ equation of the normal at $P(2,4)$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{5}(x - 2) \quad | \times 5$

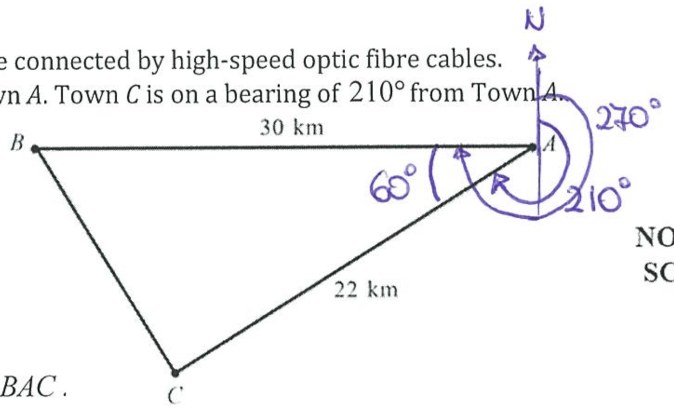
$5y - 20 = -(x - 2)$

$5y - 20 = -x + 2$

$\therefore x + 5y - 22 = 0$

√ **Question 25** (3 marks) **Marks**

Towns A, B and C are to be connected by high-speed optic fibre cables.
Town B is due west of town A. Town C is on a bearing of $210°$ from Town A.



NOT TO SCALE

(a) Find the size of $\angle BAC$. **1**

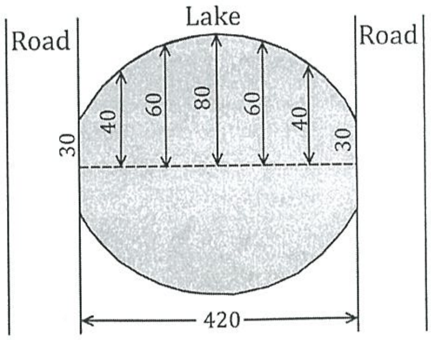
$\angle BAC = 270° - 210° = 60°$

(b) Find the distance, to the nearest kilometre, between towns B and C. **2**

$x^2 = 30^2 + 22^2 - 2 \times 30 \times 22 \times \cos(60°)$
$x^2 = 724$
$x = \sqrt{724}$
$x = 26.907...$
Distance between towns B and C is 27 km.

√ **Question 26** (3 marks)

A symmetrical lake has two roads, 420 metres apart, forming two of its sides.



Not to scale

Equally spaced measurements of the lake, in metres, are shown on the above diagram. Use the trapezoidal rule to estimate the area of the lake. **3**

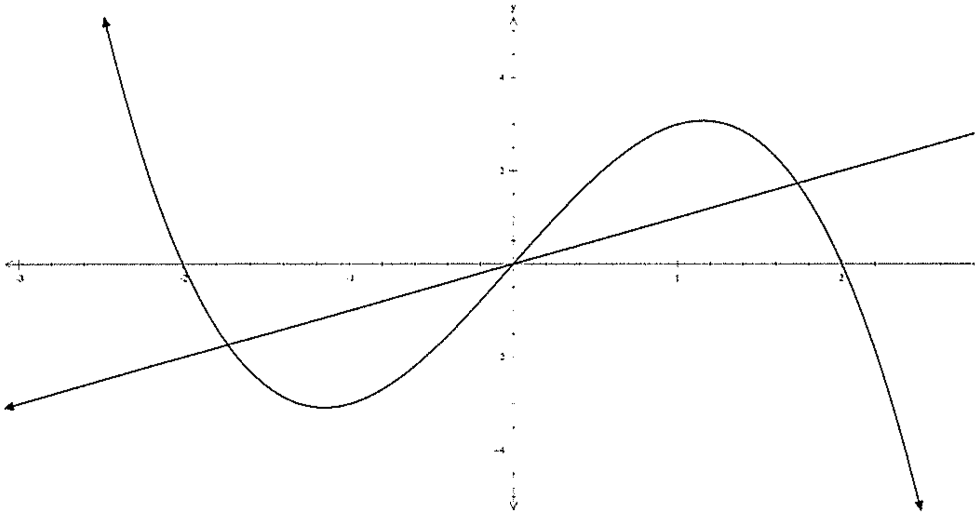
$A \doteqdot \frac{420-0}{2 \times 6}\{30 + 2 \times (40+60+80+60+40) + 30\}$

$\doteqdot 21700$ then total area of the lake is

$\doteqdot 43400\ m^2$ $(2 \times 21700\ m^2)$

Question 27 (4 marks) **Marks**

The functions $y = -x^3 + 4x$ and $y = x$ are sketched below.



(a) Show that the functions intersect when $x = 0$ and $x = \pm\sqrt{3}$. **2**

$-x^3 + 4x = x \quad | -x$

$-x^3 + 3x = 0$

$-(x^3 - 3x) = 0 \quad | \div(-1)$

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0$ or $x^2 = 3 \quad \therefore x = 0, \sqrt{3}, -\sqrt{3}$

(b) Hence find the exact area between the two functions in the first quadrant. **2**

$\int_0^{\sqrt{3}} (-x^3 + 4x)\,dx - \int_0^{\sqrt{3}} x\,dx = \int_0^{\sqrt{3}} (\underbrace{-x^3 + 4x - x}_{3x})\,dx$

$= \left[-\frac{x^4}{4} + \frac{3x^2}{2} \right]_0^{\sqrt{3}}$

$= \left(-\frac{(\sqrt{3})^4}{4} + \frac{3(\sqrt{3})^2}{2} \right) - (0 + 0)$

$= -\frac{9}{4} + \frac{9}{2} = \frac{9}{4}$

$\therefore$ Area is $\frac{9}{4}$ units^2

Question 28 (3 marks)

The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population P of a nest of this insect grows exponentially so that: 3

$$\frac{dP}{dt} = 1200e^{0 \cdot 3t}$$

A nest of these insects had a population of 5000 after one month.
Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Answer correct to the nearest month.

$\frac{dP}{dt} = 1200e^{0.3t}$

then $P = \frac{1200e^{0.3t}}{0.3} + C$

$P = 4000e^{0.3t} + C$

given: $P = 5000$ when $t = 1$:

$5000 = 4000e^{0.3\times 1} + C \quad | -4000e^{0.3}$

$C = -399.435...$

$\doteqdot -399.4$

$\therefore P = 4000e^{0.3t} - 399.435...$

→ to find t when $P = 100000$

$100000 = 4000e^{0.3t} - 399.435... \quad | +399.435 \quad | \div 4000$

$e^{0.3t} = 24.9001... \quad | \ln$

$0.3t = \ln 24.9001... \quad | \div 0.3$

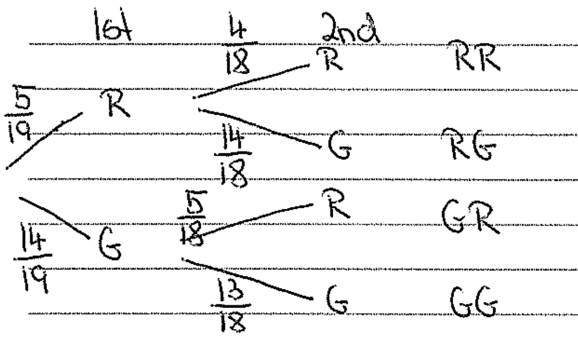
$t = 10.7162$

$\doteqdot 11$

Question 29 (3 marks)

A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.

(a) Draw a tree diagram to show the possible outcomes. Include the probability on each branch. 2



(b) What is the probability that the two lollies are of different colours? RG or GR 1

$P(\text{diff. colours}) = P(RG) + P(GR)$

$= \frac{5}{19} \times \frac{4}{18} + \frac{14}{19} \times \frac{5}{18}$

$= \frac{70}{342} + \frac{70}{342} = \frac{70}{171}$

Question 30 (7 marks) **Marks**

A particle moves along a straight line so that its displacement, x metres, from a fixed point O is given by $x = 1 + 3cos2t$, where t is measured in seconds.

(a) What is the initial displacement of the particle? **1**

$x = 1 + 3\cos 2t$

When $t = 0$: $x = 1 + 3\cos 0$

$= 1 + 3 \times 1$

$= 4$ ∴ initial displacement is 4m →.

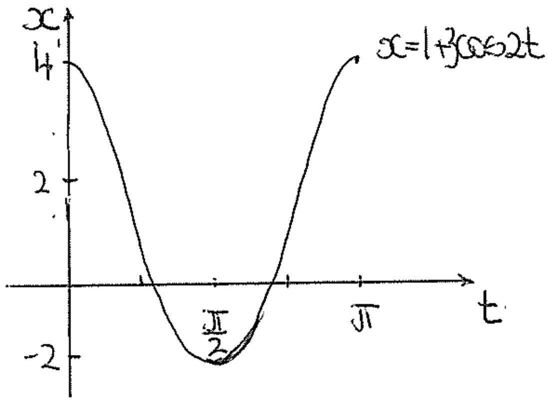
(b) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

Period: $\frac{2\pi}{n} \Rightarrow \frac{2\pi}{2} = \pi$

$-1 \leq \cos 2t \leq 1$ | ×3

$-3 \leq 3\cos 2t \leq 3$ | +1

$-2 \leq 1 + 3\cos 2t \leq 4$



(c) Hence, or otherwise, find when AND where the particle first comes to rest after $t = 0$. **2**

$x = 1 + 3\cos 2t$

$v = -3\sin(2t) \times 2$

$= -6\sin(2t)$

at rest when $v = 0$

$-6\sin 2t = 0$

$\sin 2t = 0$

$2t = 0, \pi, 2\pi, \ldots$

→ $t = 0, \frac{\pi}{2}, \pi$

After $t = 0$, particle comes first to rest when $t = \frac{\pi}{2}$. ✓

→ $x = 1 + 3\cos 2\left(\frac{\pi}{2}\right)$

$= 1 + 3\cos \pi$

$= 1 + 3 \times (-1) = -2$

∴ Particle comes to rest 2m ← ✓ to the left from the origin

(d) Find a time when the particle reaches its greatest magnitude of velocity. What is this velocity? **2**

$v = -6\sin 2t$

$a = -12\cos 2t$

greatest magn. occurs when $a = 0$:

$-12\cos 2t = 0$ | ÷(-12)

$\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}$

max. ∴ at $t = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$

→ $t = \frac{\pi}{4}$:

$v = -6\sin 2\left(\frac{\pi}{4}\right)$

$= -6\sin\frac{\pi}{2}$

$= -6 \times 1$

max. is 6 m/s^{-1}

Question 31 (2 marks) **Marks**

Find all solutions of $2sin^2x + cosx - 2 = 0$, where $0 \le x \le 2\pi$. 2

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 x = (1 - \cos^2 x)$

Sub.: $2(1-\cos^2 x) + \cos x - 2 = 0$

$2 - 2\cos^2 x + \cos x - 2 = 0$

$-2\cos^2 x + \cos x = 0$

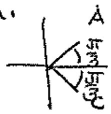
$\cos x(-2\cos x + 1) = 0$

$\cos x = 0$ or $-2\cos x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \cos x = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

→ cos ⊕ in 1st and 4th quad.



then $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$

Question 32 (3 marks)

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231 a)
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986 b)	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

(a) Find the present value, if $3200 is contributed per month for 46 months at 0.75% per month. Answer to the nearest cent. 1

$PV = 3200 \times 38.78231$

$= 124\,103.392$

$\doteqdot \$124\,103.39$

(b) Annabelle borrows $27 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 7.8% per annum interest. Find Annabelle's monthly repayment. Answer to the nearest cent. 2

$n = 4 \times 12 = 48$

$r = 7.8\% \div 12$

$= 0.65\%$

$= 0.0065$

table value $= 41.11986$

→ monthly repayment will be x

$PV = 41.11986 \times x$

$27000 = 41.11986 \times x$

$x = \frac{27000}{41.11986} = 656.6170\ldots$

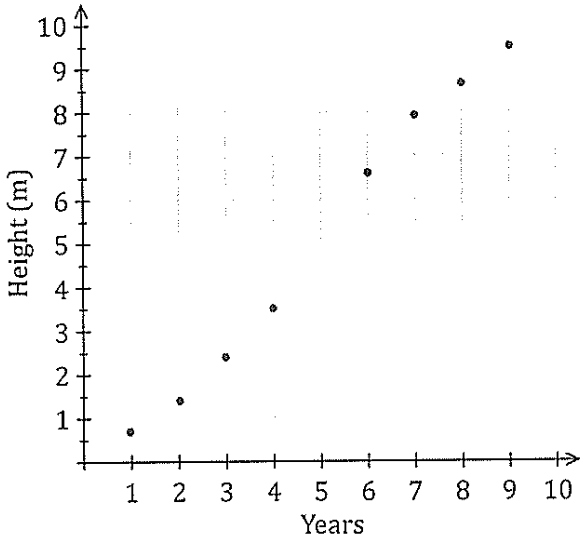
$\doteqdot \$656.62$ monthly repayment

✓ **Question 33** (6 marks) **Marks**

Hayden is an agricultural scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5		6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) What is Pearson's correlation coefficient? Answer correct to 4 decimal places. **1**

$r = 0.995193611 \doteq 0.9952$ (calculator)

(b) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). Answer correct to 2 decimal places. **2**

$y = mx + b$

$h = BX + A$ $\therefore h = 1.19t - 0.85$ (calculator)

(c) Hayden did not record the tree's height after five years. Predict the height after five years, correct to one decimal place. **1**

when $t = 5$: $h = 1.19 \times 5 - 0.85$

$\doteq 5.1$ m

(d) Use algebra to estimate how many years it will take for the tree to reach a height of 20 metres. Answer correct to 1 decimal place. **1**

$h = 20$ find t $20 = 1.19t - 0.85$

$1.19t = 20.85$ $\div 1.19$

$t \doteq 17.5$ years

(e) Comment on the reliability of your answers in (c) and (d). **1**

strong positive association

Q33c) involves interpolation (very reliable)

Q33d) involves extrapolation (less reliable)

Question 34 (6 marks) **Marks**

The height $h(t)$ metres of the tide above the mean sea level on 1st April is given by the following rule:

$$h(t) = 4\sin\left(\frac{\pi}{8}t\right)$$

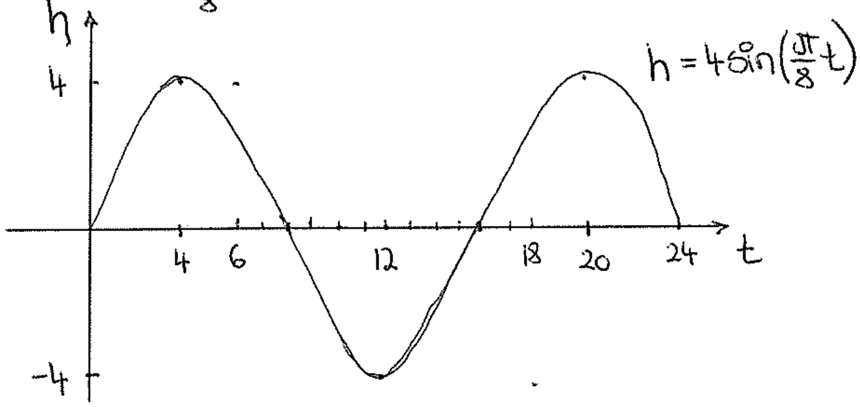
where t is the number of hours after midnight

(a) Draw a graph of $y = h(t)$ for $0 \leq t \leq 24$. 2

amplitude = 4

period = $\frac{2\pi}{n}$; $\frac{2\pi}{\frac{\pi}{8}} = 16$

24 hours = 16 + 8



(b) When was high tide? 1

when h = 4 ; t = 4 or t = 20

(c) What was the height of the high tide? 1

4 m above mean height

(d) What was the height of the tide at 10 a.m.? 2
Answer correct to one decimal place

$h(10) = 4\sin\left(\frac{\pi}{8} \times 10\right)$

$= 4\sin\left(\frac{5\pi}{4}\right)$

$= -2.8284\ldots$

$\doteqdot -2.8$ 2.8 m below mean height

End of paper