Killara
HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2020 Year 12

Mathematics Advanced

Trial Examination

17th August, 2020

Q	Marks
MC	/10
11	
12	
13	
14	
15	
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30	
31	
Total	

General Instructions:

- Reading time – 10 minutes
- Working time – 180 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I - 10 marks

- Allow about 15 minutes for this section

Section II - 90 marks

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 30% of the course.

Section I

10 marks

Allow about 15 minutes for this section

Use the multiple-choice sheet for Questions 1–10.

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1 The 7th term of an arithmetic sequence is 45 and the 11th term is 77.

Find the first term (a) and the common difference (d).

(A) $a = -3$ and $d = 8$

(B) $a = 3$ and $d = 8$

(C) $a = 8$ and $d = -3$

(D) $a = 8$ and $d = 3$

2 The function $y = \ln x$ is transformed by first being dilated vertically by a scale factor of 3 and then translated horizontally 4 units to the left.

Find the equation of the transformed function.

(A) $y = 3\ln x^4$

(B) $y = 4\ln x^3$

(C) $y = 3\ln(x - 4)$

(D) $y = 3\ln(x + 4)$

3 The solutions to $|3x-2| \geq 4$ are

(A) $x \leq 2, \quad x \leq -\frac{2}{3}$

(B) $x \leq 2, \quad x \geq -\frac{2}{3}$

(C) $x \geq 2, \quad x \geq -\frac{2}{3}$

(D) $x \geq 2, \quad x \leq -\frac{2}{3}$

4 The radius and centre of a circle with equation $x^2-6x+y^2+10y+18=0$ is

(A) Radius $= 3\sqrt{2}$, Centre $(-3,5)$

(B) Radius $= 3\sqrt{2}$, Centre $(3,-5)$

(C) Radius $= 4$, Centre $(-3,5)$

(D) Radius $= 4$, Centre $(3,-5)$

5 If $\tan\theta = \frac{2}{3}$ and θ is acute, find the exact value of $\cos\theta$.

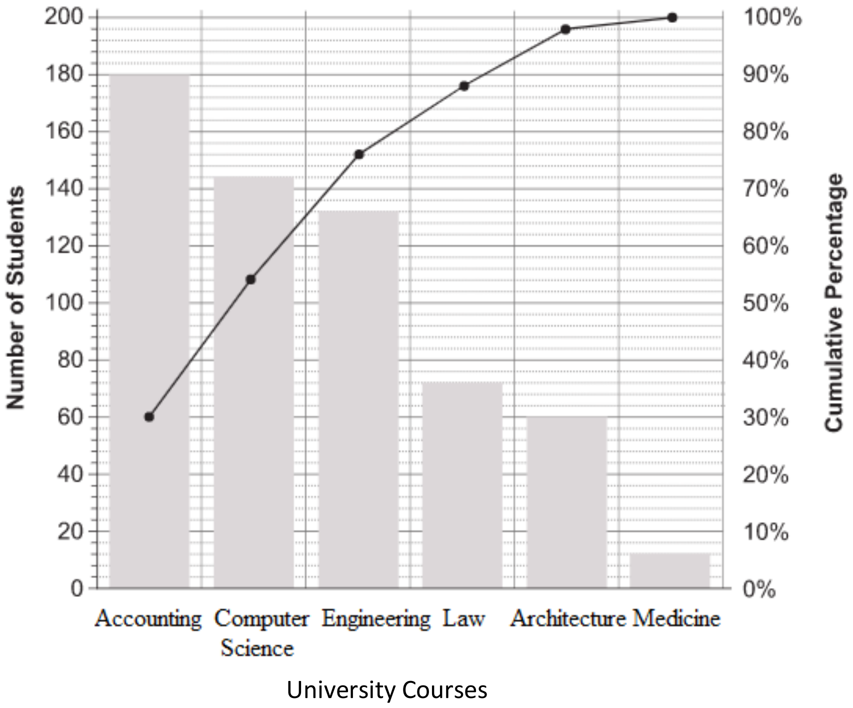
(A) $\frac{2}{\sqrt{5}}$

(B) $\frac{3}{\sqrt{5}}$

(C) $\frac{3}{\sqrt{13}}$

(D) $\frac{2}{\sqrt{13}}$

6 The Year 12 students from Koolara High School were asked which course they were interested in studying after graduating. The results from this survey are shown below.



What was the total number of students who were surveyed?

(A) 180

(B) 200

(C) 300

(D) 600

7 For what values of *x* is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

8 Let $f(x) = \frac{1}{x}$ and $g(x) = x + 2$. The domain of $f\big(g(x)\big)$ is

(A) All real x

(B) $x \neq -2$

(C) $x \neq 0$

(D) $x \neq 2$

9 Consider the set of coordinates below

$$(2,3), (5,7), (-6,2), (-8,3), (1,9)$$

This set of coordinates represent a relationship that is

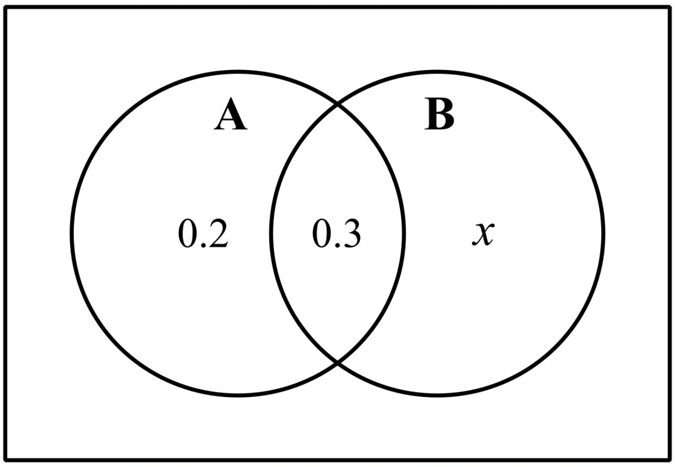
(A) One-to-one

(B) One-to-many

(C) Many-to-one

(D) Many-to-many

10 Consider the following Venn diagram.



Find the value of x such that A and B are independent events.

(A) 0.2

(B) 0.3

(C) 0.5

(D) 0.6

End of Section I

Section II

In Questions 11 – 31, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Differentiate the following with respect to *x*. **2**

$$\frac{\sin x}{x^2}$$

Question 12 (2 marks)

Determine whether the function $f(x) = \frac{x^3}{x^2-1}$ is odd, even or neither. **2**

Question 13 (4 marks)

The random variable X has this probability distribution.

$\boldsymbol{x}$	0	1	2	3	4
$\boldsymbol{P(X = x)}$	0.1	0.2	0.4	0.2	0.1

(a) Find $P(1 < X \leq 3)$. **1**

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(b) Find $E(X)$. **1**

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(c) Find the variance of X. **2**

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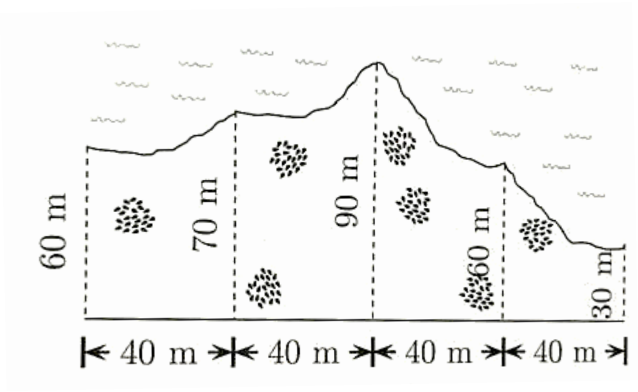
Question 14 (3 marks)

The diagram shows the land that Jane bought near a lake.



(a) Use the trapezoidal rule with 5 function values to find an estimate for the area of this land. **2**

(b) Is the area obtained in the previous part smaller or larger than the exact area? Give reasons for your answer. **1**

Question 15 (2 marks)

Koolara High School wants to sell a new sports shirt to its students at $40 each. To manufacture the shirts, the schools pays a fixed cost of $595, as well as $33 per shirt. By writing a pair of simultaneous equations, calculate how many shirts the school must sell in order to break-even. **2**

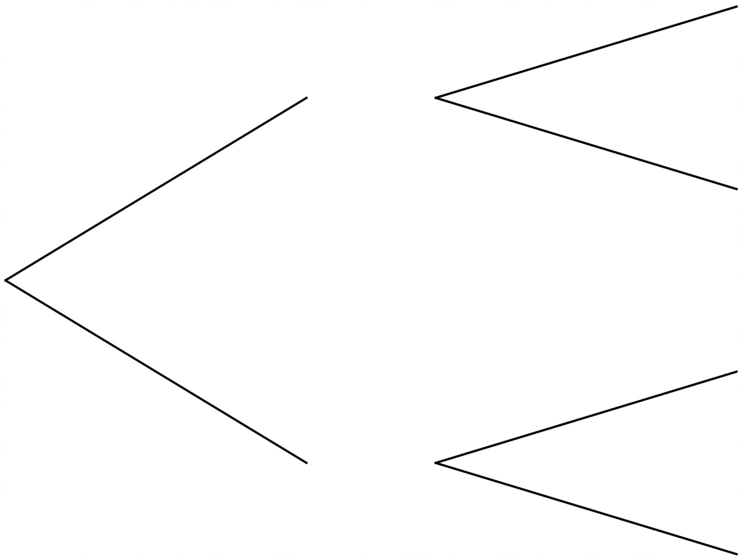
Question 16 (2 marks)

Given that $\log_a b = 3.14$ and $\log_a c = 2.45$, find the value of $\log_a \sqrt{bc}$. **2**

Question 17 (4 marks)

200 tickets are sold in a raffle at Koolara High School. First prize is $150 and second prize is $50. The prize-winning tickets are drawn consecutively without replacement where the first ticket wins first prize.

(a) Complete the probability tree diagram for a student who buys five tickets, showing the sample space clearly. **2**



(b) Find the probability that the student wins at least one prize. **1**

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(c) What is the probability that the student wins the second prize given that they did not win the first prize? **1**

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Examination continues on next page.

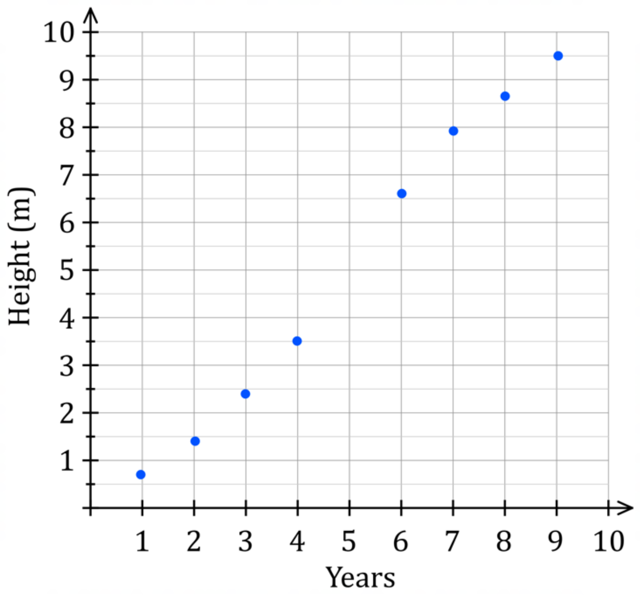
Question 18 (4 marks)

Hayden is an agricultural scientist studying the growth of a particular tree over several years.

The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, h metres	0.7	1.4	2.4	3.5	[redacted]	6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) Find the equation of the least-squares line of best fit in terms of years (t) and height (h). **1**

(b) Hayden did not record the tree's height after five years. Predict the height after five years, correct to one decimal place. **1**

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(c) Comment on the reliability of your answers in (b). **1**

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(d) Using the equation that you have determined in part (a), interpret the slope of the least-squares regression line in terms of the variables height and years. **1**

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Examination continues on next page.

Question 19 (5 marks)

A swimming pool is to be emptied for maintenance. The quantity of water, Q litres, remaining in the pool at a time, t minutes after it starts to drain, is given by:

$$Q(t) = 2000(25 - t)^2, \quad t \geq 0$$

(a) How long will it take to remove half of the water from the pool to the nearest minute? **3**

(b) At what time does the rate of flow of water from the pool reach 20 000 L/minute? **2**

Question 20 (2 marks)

Show **2**

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Examination continues on next page.

Question 21 (4 marks)

The Year 12 students at Koolara High School were surveyed on the number of hours of sleep they get per week. The results are normally distributed. The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week. **1**

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(b) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **1**

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(c) The Year 12 students at a nearby school, Chetswood High, were also surveyed. The results are also normally distributed with a mean of 50 hours and standard deviation of 2 hours. **2**

Chetswood High boast they have a greater percentage of Year 12 students that get more than 54 hours of sleep per week. Are they correct? Justify your answer with relevant calculations.

Examination continues on next page.

Question 22 (8 marks)

A particle is moving in a straight line with velocity $v = 3e^{t} + 6e^{-t}$ with t measured in minutes and v in ms^{-1}.

The particle begins its motion at the origin.

(a) What is the initial velocity? **1**

(b) Find an equation for x, the displacement of the particle. **2**

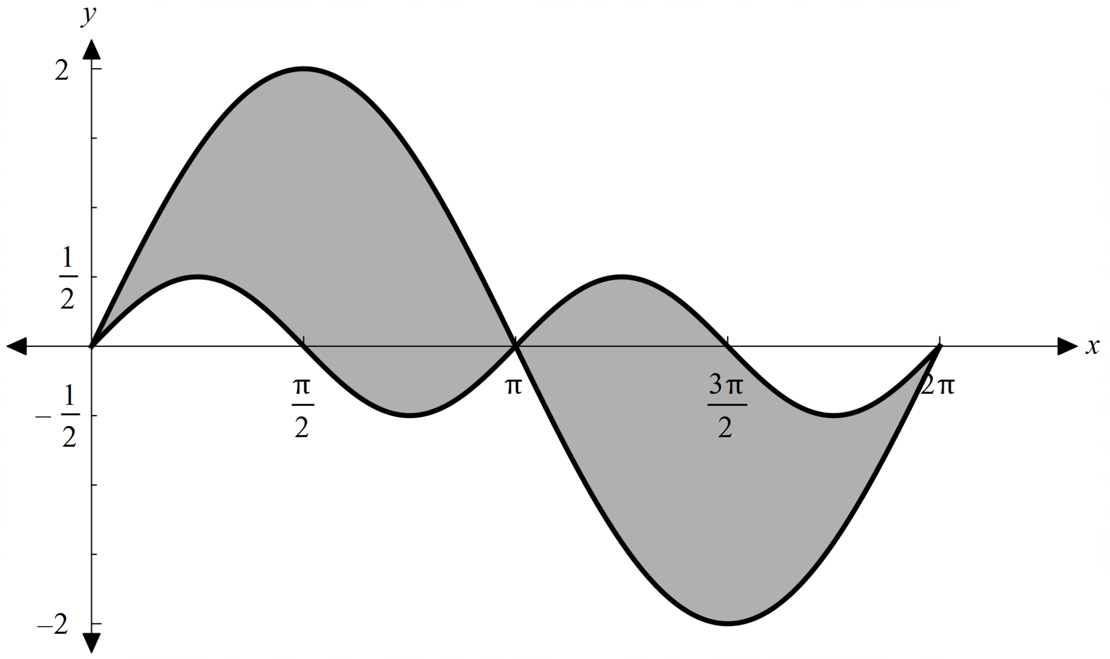
(c) Show that when $x = 10$, $3e^{2t} - 7e^{t} - 6 = 0$. **2**

(d) Hence, find the value of t when $x = 10$. **3**

Question 23 (5 marks)

An electronics company is designing a new logo, based initially on the graphs of the functions $f(x) = 2\sin(x)$ and $g(x) = \frac{1}{2}\sin(2x)$, for $0 \leq x \leq 2\pi$.

These graphs are shown in the diagram below, in which the measurements in the x and y directions are in metres.



The logo is to be painted on to a large sign, with the area enclosed by the graphs of the two functions (shaded in the diagram) to be painted red.

(a) The total area of the shaded regions, in square metres, can be calculated as **1**

$$a\int_0^{\pi} \sin(x)\, dx$$

What is the value of a?

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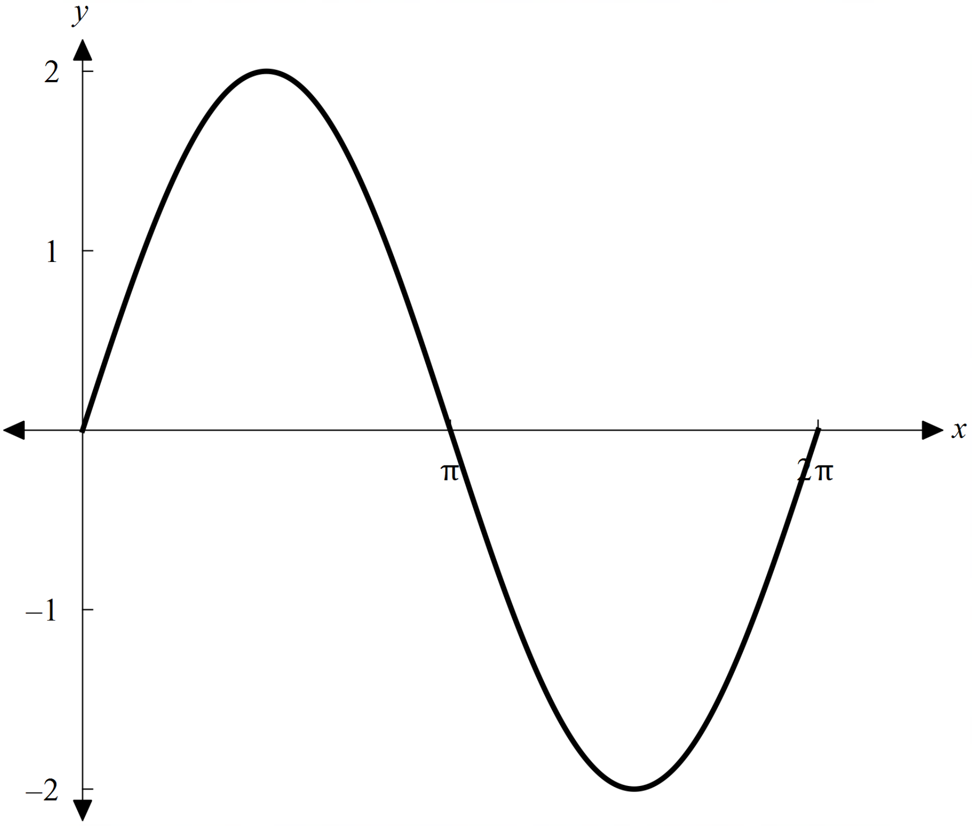
The electronics company considers changing one of the trigonometric functions used in the design of the logo. Their next attempt uses the graphs of the functions $f(x) = 2\sin(x)$ and $h(x) = \frac{1}{3}\sin(3x)$, for $0 \leq x \leq 2\pi$.

(b) On the axes below, the graph of $y = f(x)$ has been drawn. On the same axes, draw the graph of $y = h(x)$. **2**



(c) Describe in words the two transformations that maps the graph of $y = f(x)$ to the graph of $y = h(x)$. **2**

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Questions 11-23 are worth 47 marks in total

Question 24 (6 marks)

The queueing time, X minutes, of a customer at the checkout of a supermarket has the probability density function

$$f(x) = \begin{cases} \dfrac{12}{625}x^2(k - x) & 0 \le x \le k \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is 5. **3**

(b) Is a customer more likely to wait less than 3 minutes, or more than 3 minutes? Justify your answer with calculations. **3**

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Examination continues on next page.

Question 25 (8 marks)

For the curve $y = x^3 + 6x^2 + 9x$,

(a) Find any stationary points and determine their nature. **3**

(b) Sketch the curve, showing all main features, including intercepts, stationary points and any points of inflection. Use the lines below for any relevant working. **5**

Question 26 (3 marks)

The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population P of a nest of this insect grows exponentially so that: **3**

$$\frac{dP}{dt} = 1200e^{0\cdot 3t}$$

where t is in months.

A nest of these insects had a population of 5000 after one month.

Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Answer correct to the nearest month.

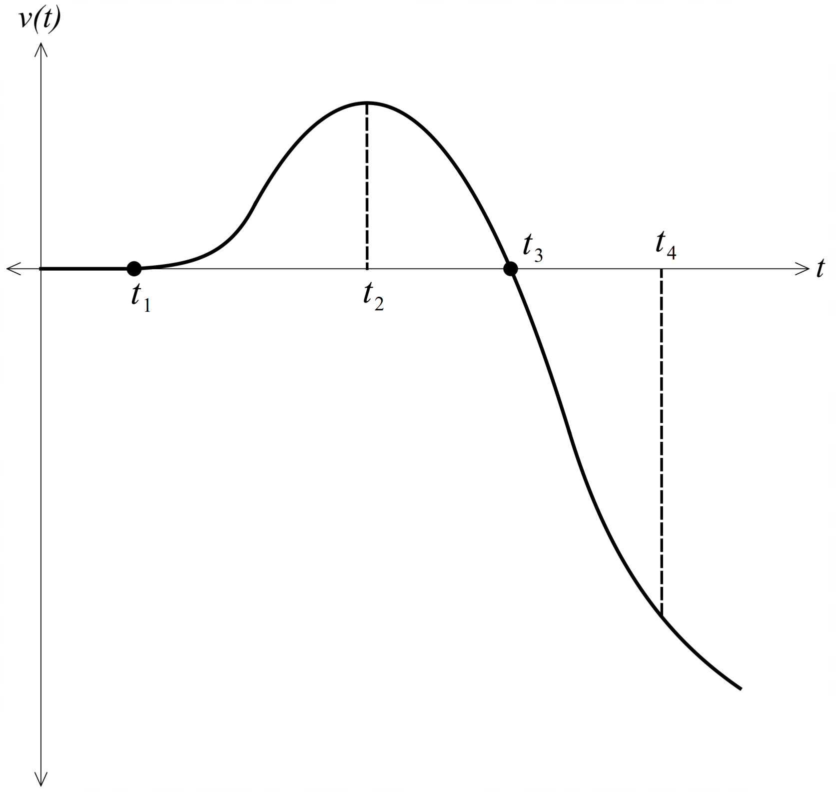
Question 27 (3 marks)

Solve $2\cos\left[2\left(x-\frac{\pi}{6}\right)\right] = 1$ in the domain $[0, 2\pi]$. **3**

Examination continues on next page.

Question 28 (7 marks)

A particle moves in a straight line and is initially 10 metres right of the origin. The velocity-time graph shown below describes this motion.



(a) What is the displacement of the particle at t_1 seconds? **1**

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(b) When is the particle at rest? **2**

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(c) When is the particle farthest to the right of the origin? **1**

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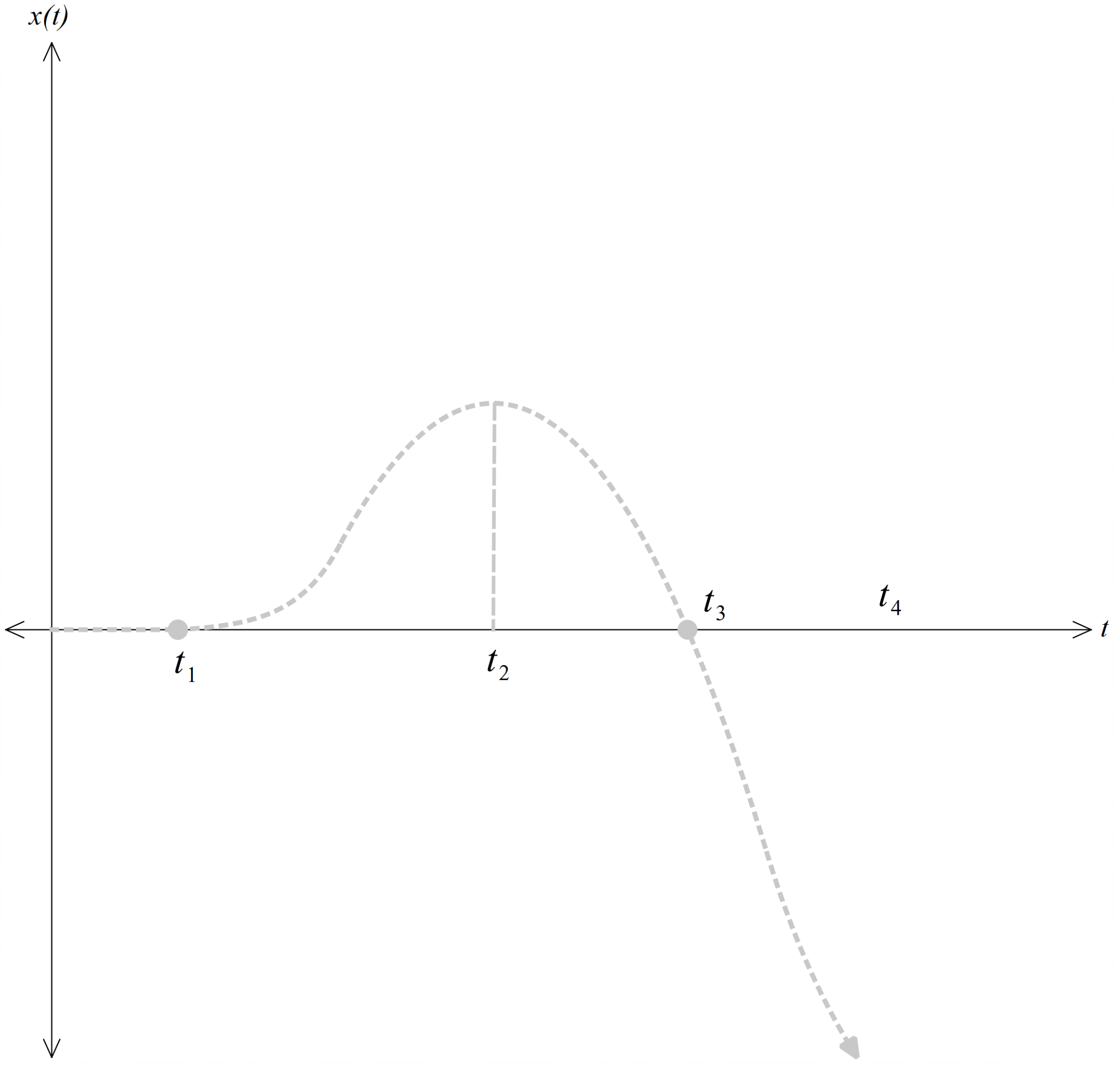
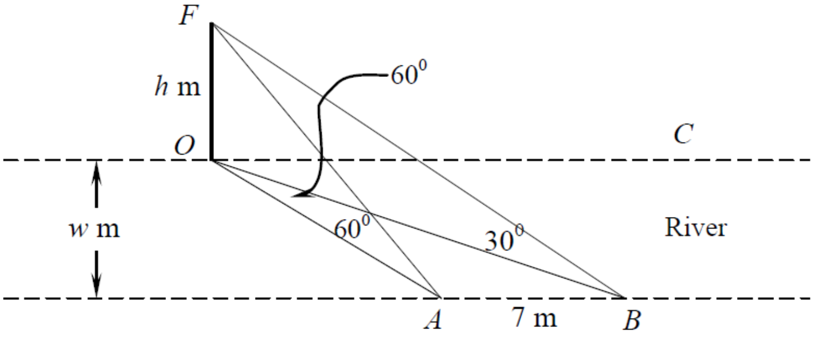
(d) Sketch the displacement-time graph. **3**



Question 29 (5 marks)

A river has level parallel riverbanks OC and AB of width w metres. OF is a vertical flagpole of height h metres which stands with its base O on the edge of riverbank OC. Positions A and B are two points on the other riverbank such that $AB = 7$ metres and $\angle AOB = 60°$.

The angle of elevation to the top of the flagpole form A and B are $60°$ and $30°$ respectively, as shown below.



(a) Show that the height of the flagpole is $\sqrt{21}$ metres. **3**

(b) By finding the area of ΔAOB, or otherwise, find the width of the river. **2**

Examination continues on next page.

Question 30 (4 marks)

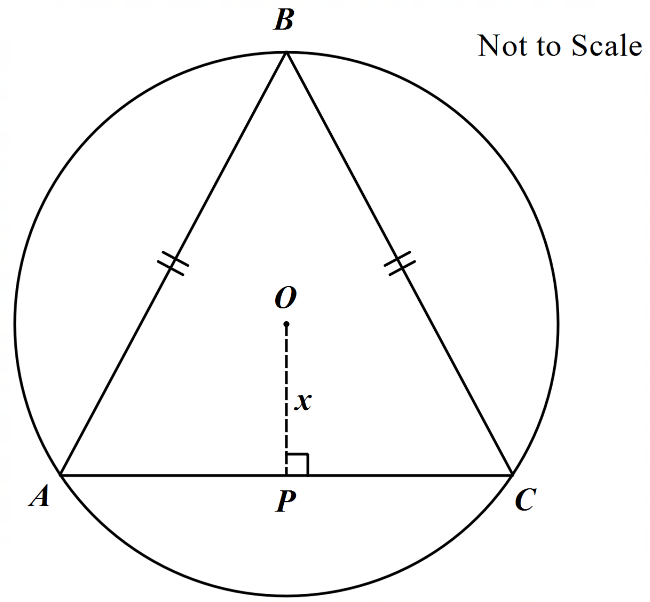
(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$. **2**

(b) Hence, or otherwise, find $\int \frac{1}{9-x^2}\,dx$. **2**

Question 31 (7 marks)

An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10 cm.

$OP = x$ and OP bisects AC, such that $AC \perp OP$.



(a) Show that the area, A, of ΔABC is given by $A = (10 + x)\sqrt{100 - x^2}$ **2**

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Examination continues on next page.

(b) By first showing that the derivative is **5**

$$\frac{dA}{dx} = \frac{100 - 10x - 2x^2}{\sqrt{100 - x^2}}$$

prove that the triangle with maximum area is equilateral.

Extra writing space

If you use this space, clearly indicate which question you are answering.

Multiple Choice Answers

1	A
2	D
3	D
4	D
5	C
6	D
7	A
8	B
9	C
10	B

Question 11 (2 marks)

Differentiate the following with respect to *x.*

$$\frac{\sin x}{x^2}$$

$u = \sin x\,,\quad v = x^2$ $u' = \cos x\,,\quad v' = 2x$ $f'(x) = \frac{vu' - uv'}{v^2}$ $= \frac{x^2 \cos x - \sin x \cdot 2x}{x^4}$	2 marks: correct solution from correct working 1 mark: attempt to use the quotient rule or attempt to use the product rule with x^{-2}	Students who used the quotient rule were more successful than those who used the product rule. Many students did not simplify fully.

Question 12 (2 marks)

Determine whether the function $f(x) = \frac{x^3}{x^2-1}$ is odd, even or neither.

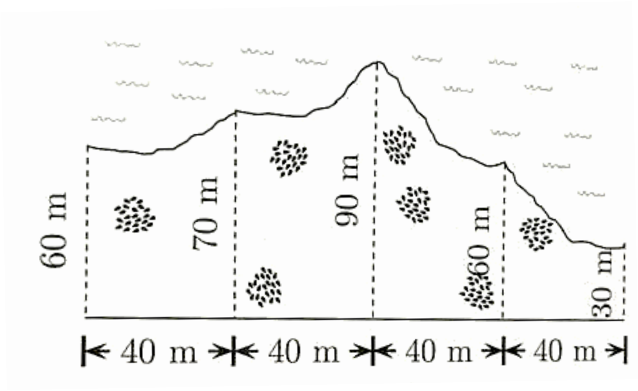
$f(-x) = \frac{(-x)^3}{(-x)^2 - 1}$ $= -\frac{x^3}{x^2 - 1}$ $= -f(x)$ $\therefore f(x)$ is an odd function	2 marks: correctly shown $f(x)$ is odd by showing $f(-x) = f(x)$ 1 mark: attempts to find $f(-x)$	Many students got this correct by showing f(-x) = f(x)

Question 13 (4 marks)

The random variable *X* has this probability distribution.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1
$(x - \mu)^2$	4	1	0	1	4
$(x - \mu)^2 p(x)$	0.4	0.2	0	0.2	0.4

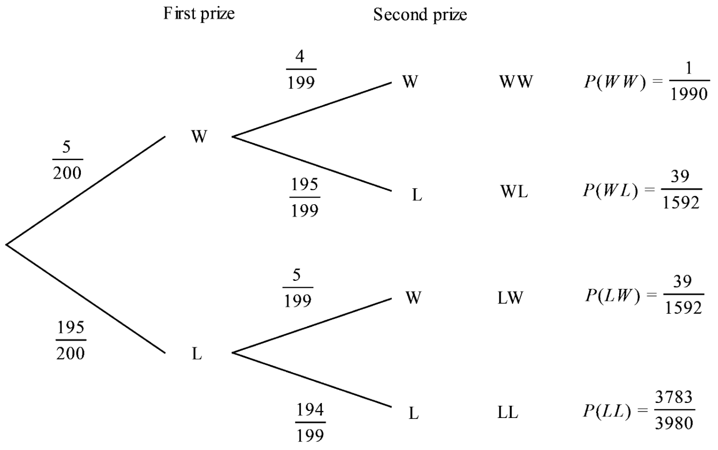
(a) Find $P(1 < X \leq 3)$ $P(1 < X \leq 3) = P(X = 2) + P(X = 3)$ $= 0.4 + 0.2$ $= 0.6$	1 mark: correct solution is given	Most students answered this question correctly, however some students included the probability of getting a 1.
(b) Find $E(X)$ $\mu = E(X)$ $= 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.4 + 3 \times 0.2 + 4 \times 0.1$ $= 2$	1 mark: correct solution is given with working. This can include adding a '$xp(x)$' column underneath the table given	Most students answered this question correctly, however there were a few calculators errors amongst those students who got the incorrect answer.
(c) Find the variance of X. $Var(X) = \Sigma(X - \mu)^2 p(x)$ $= 0.4 + 0.2 + 0.2 + 0.4$ $= 1.2$ Or $Var(X) = \sum x^2 p(x) - \mu^2$ can be used	Method 1: 2 marks: correct solution and has shown $E((x - \mu)^2)$, which can be given as working under the table provided. 1 mark: attempted to find $(x - \mu)^2$ Method 2:	

	2 marks: correct solution by finding $E(X^2) - \mu^2$ first 1 marks: attempted to find $E(X^2)$	
Question 14 (3 marks) The diagram shows the land that Jane bought near a lake.		
(a) Use the trapezoidal rule with 5 function values to find an estimate for the area of this land. $Area = \frac{40}{2}[60 + 30 + 2(70 + 90 + 60)]$ $= 20 \times (90 + 440)$ $= 10600\ m^2$	2 marks: correct application of the trapezoidal rule, with a correct answer and units 1 mark: correct application of the formula with a correct answer, but no units OR Attempt to use the trapezoidal rule with a minor error, and units are given	Many students made mistakes in finding the values for $\frac{b-a}{n}$ and a handful of students forgot to include units in their answer.
(b) Is the area obtained in the previous part smaller or larger than the exact area? Give reasons for your answer. The area is obtained in (a) is larger than the exact area because the 4 divided exact area are all under each partitioned trapezium.	1 mark: has concluded that the area in part (a) is an over estimation, and justified by talking about concavity, or have drawn lines on the diagram and referenced them in their answer	Most students answered this correctly by drawing the trapeziums on the diagram.

<table>
<tr><td colspan="3">Question 15 (2 marks)
Koolara High School wants to sell a new sports shirt to its students at $40 each. To manufacture the shirts, the schools pays a fixed cost of $595, as well as $33 per shirt. How many shirts must the school sell in order to break-even?</td></tr>
<tr><td>Let x be the number of shirts.
Total cost for the school $= \$595 + 33x$
$595 + 33x = 40x$
$7x = 595$
$\therefore x = 85$ shirts</td><td>2 marks: has found the correct solution by solving simultaneous equations, or setting up an appropriate equation to solve
1 mark: has correctly shown an equation for either the cost or revenue given</td><td>Most students answered this question correctly by forming the two equations. Once the equations were formed, very few mistakes were made.</td></tr>
<tr><td colspan="3">Question 16 (2 marks)
Given that $\log_a b = 3.14$ and $\log_a c = 2.45$, find the value of $\log_a \sqrt{bc}$.</td></tr>
<tr><td>$\log_a \sqrt{bc} = \frac{1}{2}\log_a bc$
$= \frac{1}{2}[\log_a b + \log_a c]$
$= \frac{1}{2}(3.14 + 2.45)$
$= 2.795$</td><td>2 marks: has found the correct solution and shown the appropriate logarithmic laws
1 mark: has correctly applied one logarithmic law and a reasonable attempt has been made towards the solution</td><td>Many students forgot at least one logarithmic law. The most common forgotten law being $\sqrt{bc} = \frac{1}{2}bc$</td></tr>
</table>

Question 17 (4 marks)

200 tickets are sold in a raffle at Koolara High School. First prize is \$150 and second prize is \$50. The prize-winning tickets are drawn consecutively without replacement where the first ticket wins first prize.

(a) Complete the probability tree diagram for a student who buys five tickets, showing the sample space clearly.  	2 marks: Correct tree drawn with probabilities on the branches and sample space listed on the right. 1 mark: Correct tree drawn with probabilities on the branches but sample space not listed OR 1 mark: Correct probabilities drawn in for first prize but incorrect probabilities for second prize and sample space listed	This was generally poorly done. Most students did not attempt to write sample space, despite the question asking for it. Many students managed to draw the correct first branch did not realise that these were dependent events, and did not correctly fill the second branches. Students should also practice filling out tree diagrams. The probabilities go on the branches, and the outcome should be at the end of branches.
(b) Find the probability that the student wins at least one prize. $P(wins\ at\ least\ one\ prize) = 1 - P(LL)$ $= 1 - \frac{3783}{3980}$ $= \frac{197}{3980}$	1 mark: correct answer with correct working, either using the complement or adding the probabilities together	Many students did this correctly, or had the correct process from an incorrect diagram

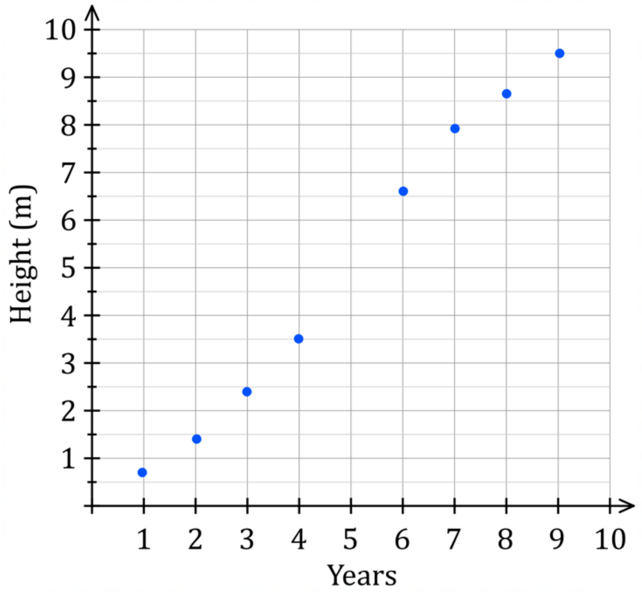
(c) What is the probability that the student wins the second prize given that they did not win the first prize? $P(wins\ second\ prize\ \vert did\ not\ win\ first\ prize) = \frac{5}{199}$	1 mark: correct.	Not many students got this correct. The phrase 'given that' means we are looking for P(LW|L). Most students just found P(LW).

Question 18 (4 marks)

Hayden is an agricultural scientist studying the growth of a particular tree over several years. The data he recorded is shown in the table below.

Years since planting, t	1	2	3	4	5	6	7	8	9
Height of tree, h metres	0.7	1.4	2.4	3.5	[redacted]	6.6	7.9	8.7	9.5

A scatterplot of the data is shown below.



(a) Find the equation of the least-squares line of best fit in terms of years (*t*) and height (*h*). Answer correct to 2 decimal places. $h = -0.85 + 1.19t$	1 mark: correct equation is given of h in terms of t	Not many students got this correct. A calculator is needed to find the least-squares regression line, and this should be a very easy mark to get.
(b) Hayden did not record the tree's height after five years. Predict the height after five years, correct to one decimal place. $h = -0.85 + 1.19 \times 5$ $\approx 5.1\ m$	1 mark: correct solution given based on the equation shown in part (a). Working is shown to substitute in	Most students received a mark for this, by substituting 5 into their equation. Students are reminded to show their substitution, even if the answer is obvious.
(c) Comment on the reliability of your answers in (b). The answer to part (b) is reasonably reliable given that the data point we are interested in estimating within the domain of the collected data. That is to say, we are interpolating this value from the collected data.	1 mark: student has justified the reliability by referring to 'interpolation'	Not many students got this correct. Using the equation is ONLY reliable when we are interpolating data. Saying it follows the line of best fit is not enough. Students should also realise that reliability is not the same as validity. Reliability is the measure used to find the answer in part (b), which is the equation. Using a least-squares regression line will be reliable, even if your equation is wrong and the answer is not valid
(d) Using the equation that you have determined in part (a), interpret the slope of the least-squares regression line in terms of the variables height and years. As the age increases by 1 year, the height has increased by 1.19 m each year on average.	1 mark: has commented on the fact that the gradient refers to growth rate of height per year	Many students did not interpret the slope. Students needed to say what the gradient meant in the context of the question.

Question 19 (5 marks)

A swimming pool is to be emptied for maintenance. The quantity of water, *Q* litres, remaining in the pool at a time, *t* minutes after it starts to drain, is given by:

$$Q(t) = 2000(25 - t)^2, \quad t \geq 0$$

(a) How long will it take to remove half of the water from the pool to the nearest minute? $t = 0$ $Q(0) = 2000 \times 25^2$ $= 1250000\ L$ $Half\ quantity = 625000\ L$ $625000 = 2000(25 - t)^2$ $312.5 = (25 - t)^2$ $25 - t = \pm\sqrt{312.5}$ $t = 25 - \sqrt{312.5}$, since $t \leq 25$ $= 7.3223 \ldots$ ≈ 7 mins	3 marks: has correctly found the original quantity by substituting in $t = 0$, found half the original quantity, and substituted back in to correctly find the time that occurs and has correctly identified the need to reject $\sqrt{312.5}$ from the calculation for t 2 marks: has correctly found the original quantity by substituting in $t = 0$, found half the original quantity, and substituted back in, but has failed to reject the correct time. 1 mark: has found the original quantity of the pool, and then found half the quantity OR has substituted the wrong amount for $Q(t)$ and found a reasonable but incorrect time	Many students got the right answer, but received 2 marks. This was because students did not consider the positive and negative square roots when solving quadratic equations. Some students found two solutions, but rejected incorrectly.
(b) At what time does the rate of flow of water from the pool reach 20 000 L/minute? $Q'(t) = -4000(25 - t)$ $-200000 = -4000(25 - t)$ $5 = 25 - t$ $t = 20$ mins	2 marks: has correctly found the derivative and found the correct time 1 mark: has found the correct derivative of $Q(t)$ OR	Many students found the derivative correctly. However, they solved the equation Q'(t)=20,000, which is incorrect. Because the flow is outward, it needed to be negative.

	Has forgotten the negative in the derivative $(Q'(t) = 4000(25 - t))$ and has found the incorrect time using that	
Question 20 Show $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$		
$LHS = \frac{1}{(1+\sin\theta)}+\frac{1}{1-\sin\theta}$ $= \frac{1-\sin\theta}{1-\sin^2\theta}+\frac{1+\sin\theta}{1-\sin^2\theta}$ $= \frac{2}{1-\sin^2\theta}$ $= \frac{2}{\cos^2\theta}$ $= 2\sec^2\theta$ $= RHS$	2 marks: has shown the $LHS = RHS$ clearly showing the use of the Pythagorean identity 1 mark: has put the LHS over a common denominator, and has attempted to simplify but has made an error	Most students got this question correct. However, they are reminded to try and be neat with these questions, starting with LHS and showing a logical sequence to get to the RHS.

Question 21 The Year 12 students at Koolara High School were surveyed on the number of hours of sleep they get per week. The results are normally distributed. The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.		
(a) Determine the mean number of hours of sleep per week. $\frac{54-42}{4}=3$ $\sigma=3$ $\mu=48\text{ hours}$	1 mark: has found the mean number of hours, with appropriate working or with a diagram	Most students got this correct, showing some correct working.
(b) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? $P(x\geq 51)=\frac{(100\%-68\%)}{2}$ $=16\%$ $P(x\leq 57)=\frac{100\%-99.7\%}{2}$ $=0.15\%$ $P(51\leq x\leq 57)=16\%-0.15\%$ $=15.85\%$	1 mark: has found the correct percentage with appropriate working, which may be a diagram, referring the probabilities ($P(51\leq x\leq 57)$), or using z-scores.	Many students did this correctly, showing working with a diagram or otherwise.

The Year 12 students at a nearby school, Chetswood High, were also surveyed. The results are also normally distributed with a mean of 50 hours and standard deviation of 2 hours. **(c) Chetswood High boast they have a greater percentage of Year 12 students that get more than 54 hours of sleep per week. Are they correct? Justify your answer with relevant calculations.** $\mu = 50, \sigma = 2$ $z_c = \frac{54 - 50}{2}$ $= 2$ $z_k = \frac{54 - 48}{3}$ $= 2$ Since the z-score for both schools is 2, the same percentage of students in each school get more than 54 hours of sleep per week.	2 marks: has found that the z-scores are the same, and justified that the percentage are therefore the same 1 mark: has found 1 correct z-score	This was done reasonably well. A common mistake was finding the wrong percentage. It is easier to find z-scores than percentages. You also needed to show both z-scores, to compare them, and make a judgement from that.

Question 22 (8 marks)

A particle is moving in a straight line with velocity $v = 3e^{t} + 6e^{-t}$ with *t* measured in minutes and *v* in ms^{-1}.

The particle begins its motion at the origin.

(a) What is the initial velocity? $t = 0$ $v = 3e^{0} + 6e^{0}$ $= 3 + 6$ $= 9\ ms^{-1}$	1 mark: has found the correct velocity with correct substitution	
(b) Find an equation for *x*, the displacement of the particle. $x = \int (3e^{t} + 6e^{-t})\,dt$ $= 3e^{t} - 6e^{-t} + c$ $t = 0, x = 0$ $0 = 3e^{0} - 6e^{0} + c$ $c = 6 - 3$ $= 3$ $x = 3e^{t} - 6e^{-t} + 3$	2 marks: has found the correct anti-derivative with the correct constant, showing the correct working 1 mark: has correctly shown working to find the anti-derivative, but not the constant	
(c) Show that when *x* = 10, $3e^{2t} - 7e^{t} - 6 = 0$. $10 = 3e^{t} - 6e^{-t} + 3$ $3e^{t} - 6e^{-t} - 7 = 0$ $3e^{2t} - 7e^{t} - 6 = 0$	2 marks: has shown the equation, but substituting in and showing the multiplication by e^{t} 1 mark: has substituted in and shown the equation, but has not explicitly shown the multiplication by e^{t}	

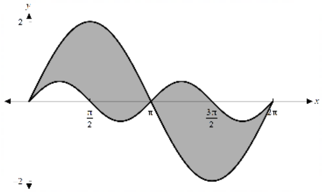
(d) Hence, find the value of *t* when *x* = 10. $3e^{2t} - 7e^{t} - 6 = 0$ $(3e^{t} + 2)(e^{t} - 3) = 0$ $e^{t} = -\frac{2}{3}$ or $e^{t} = 3$ $e^{t} = -\frac{2}{3}$ has no solution as $\mathrm{e}^{\mathrm{t}} > 0$ for all t $e^{t} = 3$ $t = \ln(3)$	3 marks: has solved the correct quadratic equation, found both expressions for t and has discarded the negative solution 2 marks: has found the correct solutions for e^{t}, and has made an error finding t, or has not discarded the negative solution 1 mark: has attempted to either substitute $u = e^{t}$, or otherwise showing an understanding that the equation reduces to a quadratic equation	

Question 23 (5 marks)

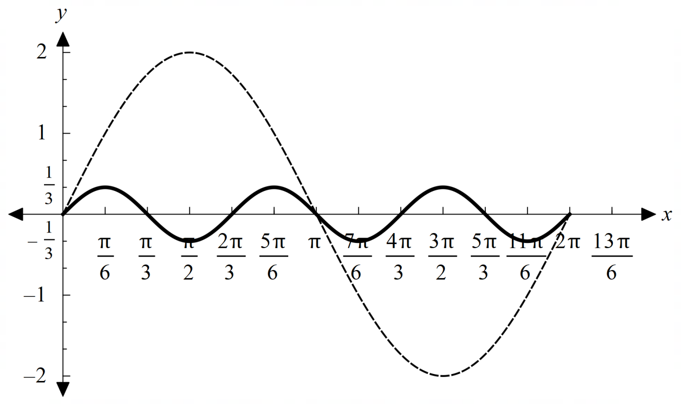
An electronics company is designing a new logo, based initially on the graphs of the functions

$f(x) = 2\sin(x)$ and $g(x) = \frac{1}{2}\sin(2x)$, for $0 \le x \le 2\pi$.

These graphs are shown in the diagram below, in which the measurements in the x and y directions are in metres.



(a) The total area of the shaded regions, in square metres, can be calculated as $$a\int_{0}^{\pi} \sin(x)\ dx$$ **What is the value of a?** $a = 4$	1 mark: stated the correct value of $a = 4$	
(b) On the axes below, the graph of $y = f(x)$ has been drawn. On the same axes, draw the graph of $y = h(x)$. $$\text{Amplitude} = \frac{1}{3}$$ $$\text{Period} = \frac{2\pi}{3}$$	2 marks: correct sketch is given, showing amplitude and intercepts/period 1 mark: has correctly shown the correct amplitude OR period in their sketch	

		
(c) Describe in words the two transformations that maps the graph of $y = f(x)$ to the graph of $y = h(x)$. Vertically stretched by factor of $\frac{1}{6}$ and horizontally stretched by a factor of $\frac{1}{3}$	2 marks: has correctly described both the horizontal dilation (1/3) and vertical dilation (1/6) 1 mark: has correctly described both the horizontal dilation or vertical dilation	

Question 24 (6 marks)

The queueing time, X minutes, of a customer at the checkout of a supermarket has the probability density function

$$f(x) = \begin{cases} \frac{12}{625}x^2(k-x) & 0 \le x \le k \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is 5. $\int_0^k \frac{12}{625}x^2(k-x)\,dx = 1$ $\frac{12}{625}\int_0^k kx^2 - x^3\,dx = 1$ $\left[\frac{kx^3}{3} - \frac{x^4}{4}\right]_0^k = \frac{625}{12}$ $\frac{k^4}{3} - \frac{k^4}{4} = \frac{625}{12}$ $\frac{k^4}{12} = \frac{625}{12}$ $k^4 = 625$ $\therefore k = \sqrt[4]{625}, k \ge 0$ $k = 5$	3 marks: has integrated correctly, and has simplified the expression correctly to show $k = 5$ and rejected $k < 0$ 2 marks: correctly integrated between 0 and k, and let the anti-derivative = 1 1 mark: has set up a correct integral, showing the area = 1	
(b) Is a customer more likely to wait less than 3 minutes, or more than 3 minutes? Justify your answer with calculations. $P(X \ge 3) = \int_3^5 \frac{12}{625}x^2(5-x)\,dx$ $= \frac{12}{625}\int_3^5 5x^2 - x^3\,dx$	3 marks: has justified why it is more likely to wait more than 3 minutes, with correct working with integration 2 marks: has correctly integrated between the appropriate bounds, and found a probability, but has not interpreted this to answer the question	

$= \frac{12}{625}\left[\frac{5x^3}{3} - \frac{x^4}{4}\right]_3^5$ $= \frac{12}{625}\left[\left(\frac{625}{3} - \frac{625}{4}\right) - \left(45 - \frac{81}{4}\right)\right]$ $= \frac{12}{625}\left[\frac{625}{12} - \frac{99}{4}\right]$ $= \frac{328}{625}$ $= 0.5248$ This means that a customer is more likely to wait more than 3 minutes.	1 mark: has attempted to integrate between 0 and 3, or 3 and 5	
Question 25 (8 marks) For the curve $y = x^3 + 6x^2 + 9x$:		
(a) Find any stationary points and determine their nature. $\frac{dy}{dx} = 3x^2 + 12x + 9$ $\text{solve} \frac{dy}{dx} = 0$ $3x^2 + 12x + 9 = 0$ $x^2 + 4x + 3 = 0$ $(x+3)(x+1) = 0$ $\therefore x = -3, x = -1$ $\frac{d^2y}{dx^2} = 6x + 12$ $\text{when } x = -3,$	3 marks: the coordinates of the turning points are found, and either the second derivative or a table of values for the first derivate is used to determine the nature of the stationary points correctly 2 marks: the coordinates of the turning points are found OR The x-coordinates of the stationary points are found, and either the second derivative or a table of values for the first derivate is used to determine the nature of the stationary points correctly	Most students were able to calculate the first derivative and find the stationary points. Students used the second derivative or first derivative table to classify the points very well. There was a large handful of students who did not write a conclusion by stating the **coordinates** of the stationary points and classifying them. This resulted in a loss of 1 mark.

<table>
<tr>
<td>$\frac{d^2y}{dx^2} = 6(-3) + 12$
$= -6 < 0$
$y = (-3)^3 + 6(-3)^2 + 9(-3)$
$= 0$
Therefore, a maximum t.p. at $(-3,0)$.
when $x = -1$,
$\frac{d^2y}{dx^2} = 6(-1) + 12$
$= 6 > 0$
$y = (-1)^3 + 6(-1)^2 + 9(-1)$
$= -4$
Therefore, a minimum t.p. at $(-1, -4)$.</td>
<td>1 mark: correct differentiation is given and the x-coordinates of the stationary points are found
OR
The incorrect x-coordinates of the stationary points are found, but, using these, either the nature of the points are determined or the coordinate of the point is found</td>
<td></td>
</tr>
<tr>
<td>(b) Sketch the curve, showing all main features, including intercepts, stationary points and any points of inflection. Use the lines below for any relevant working.
For possible points of inflection, solve $\frac{d^2y}{dx^2} = 0$
$6x + 12 = 0$
$6x = -12$
$x = -2$
<table>
<tr><td>y</td><td>-3</td><td>-2</td><td>-1</td></tr>
<tr><td>$\frac{d^2y}{dx^2}$</td><td>-6</td><td>0</td><td>6</td></tr>
<tr><td></td><td>∩</td><td>.</td><td>∪</td></tr>
</table>
</td>
<td>5 marks: a correct sketch is given, showing turning points, the point of inflection and the intercepts, and correct calculations are used the find these features

4 marks: the information for 5 marks is all given but a minor error has been made

3 marks: a correct cubic curve is given, with the stationary points and EITHER the intercepts or point of inflection is correctly shown. Correct working is also shown to support this
OR</td>
<td>Most students struggled with this question. Many students used the second derivative to find the point of inflection, but did not use the table to show that it was a point of inflection. This resulted in a loss of 1 mark.

Many students did not label the axes, had poor scale or did not include all the critical features on the graph.</td>
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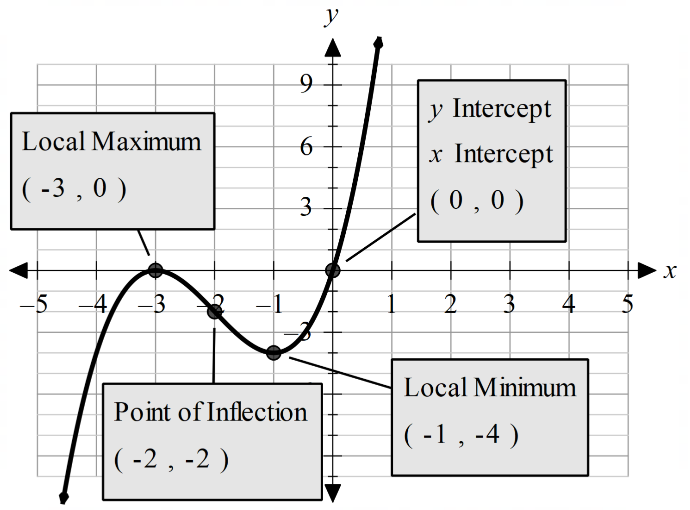
When $x = -2$,

$$y = (-2)^3 + 6(-2)^2 + 9(-2)$$
$$= -2$$

$\therefore (-2, -2)$ is a point of inflection.

For x-intercepts, solve $y = 0$

$$x^3 + 6x^2 + 9x = 0$$
$$x(x^2 + 6x + 9) = 0$$
$$x(x + 3)^2 = 0$$
$$x = 0, x = -3$$



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Several mistakes have been made in finding the point of inflection and/or intercepts, but these mistakes are reflected correctly with the graph

2 marks: a cubic function with the correct turning points is sketched, and an attempt has been made to find the point of inflection or intercepts

OR

The correct working is given the find the intercepts and points of inflection, but no graph is attempted

OR

Either a correct point of inflection or intercepts have been found with relevant working, and this is shown on the graph with a cubic function

1 mark:

A cubic function with the stationary points is sketched

OR

The x-coordinate of the point of inflection is correctly found, with no sketch

OR

The x-intecepts are correctly found, with no correct sketch

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Question 26 (3 marks)

The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population *P* of a nest of this insect grows exponentially so that:

$$\frac{dP}{dt} = 1200e^{0\cdot 3t}$$

where t is in months.

A nest of these insects had a population of 5000 after one month.

Determine how long it will take the nest to reach the viable stage (i.e. when the population has reached 100 000). Answer correct to the nearest month.

$P = \int 1200e^{0.3t}\,dt$ $= 1200 \times \frac{1}{0.3}e^{0.3t} + c$ $= 4000e^{0.3t} + c$ When $t = 1, P = 5000$ $5000 = 4000e^{0.3} + c$ $c = 5000 - 4000e^{0.3}$ $c = -399.435 \ldots$ $P = 4000e^{0.3t} - 399.435 \ldots$ when $P = 100\,000,$ $100000 = 4000e^{0.3t} - 399.435 \ldots$ $100399.435 \ldots = 4000e^{0.3t}$ $25.0998 \ldots = e^{0.3t}$ $\ln(25.0998 \ldots) = 0.3t$ $t = \frac{\ln(25.0998)}{0.3}$ $t = 10.7482 \ldots$ $\approx 11 \text{ months}$	3 marks: A correct anti-derivate with the constant is found, and 100,000 is shown to be substituted in to find the correct value of t 2 marks: the anti-derivate and the constant are found, and 100,00 has been attempted to be substituted in, but the correct value of t has not been found OR the anti-derivate but not the constant has been found, and 100,00 has been substituted in to find t 1 mark: the anti-derivate is correctly found, without the constant OR A reasonable but incorrect anti-derivate is found and 100,000 is substituted in	There was a frightening number of students who e as a pronumeral rather than a number. This meant when they were solving for the constant c, they applied the incorrect inverse operations to solve for c. Otherwise, this question was mostly done well.

Question 27 (3 marks)

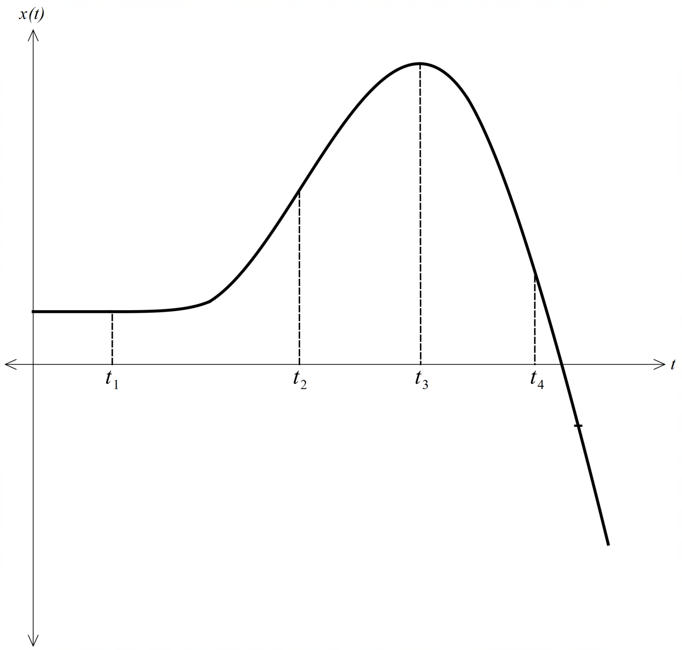
Solve $2\cos\left[2\left(x-\frac{\pi}{6}\right)\right] = 1$ in the domain $[0, 2\pi]$.

$\cos\left[2\left(x-\frac{\pi}{6}\right)\right]=\frac{1}{2}$ $0 \le x \le 2\pi$ $-\frac{\pi}{6} \le x-\frac{\pi}{6} \le \frac{11\pi}{6}$ $-\frac{\pi}{3} \le 2\left(x-\frac{\pi}{6}\right) \le \frac{11\pi}{3}$ $2\left(x-\frac{\pi}{6}\right) = -\frac{\pi}{3},\frac{\pi}{3},2\pi-\frac{\pi}{3},2\pi+\frac{\pi}{3},4\pi-\frac{\pi}{3}$ $2\left(x-\frac{\pi}{6}\right) = -\frac{\pi}{3},\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3}$ $x-\frac{\pi}{6} = -\frac{\pi}{6},\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ $x = 0,\frac{\pi}{3},\pi,\frac{4\pi}{3},2\pi$	3 marks: 2 marks: correct working (substitution or otherwise) is given to find three correct solutions $(2x = 0, \frac{2\pi}{3}, 2\pi)$ 1 mark: one correct solution is found with relevant working (guess and check) OR A substitution of $u = 2\left(x-\frac{1}{6}\right)$ is used to show $u = \cos^{-1}\left(\frac{1}{2}\right)$ or equivalent	This question was done poorly. There is a huge proportion of students who need to carefully revise their skills in solving trigonometric equations. Those who made a good attempt often forgot that *x=0* is also a solution.

Question 28 (7 marks)

A particle moves in a straight line and is initially 10 metres right of the origin. The velocity-time graph shown below describes this motion.

(a) What is the displacement of the particle at t_1 seconds? 10 metres to the right of the origin.	1 mark: correct answer given	This question was done surprisingly poorly. Most students forgot to state “to the right of the origin”.
(b) When is the particle at rest? The particle is at rest from $0 \leq t \leq t_1$ and at $t = t_3$	2 marks: given inequalities $0 \leq t \leq t_1$ and $t = t_3$ 1 mark: one of the above correct solutions	Most students were able to identify t=t3 as a solution, but many stated t=t1 as a solution and this is incorrect. Most students failed to recognise that 0 ≤ t ≤ t1is a solution. There were a handful of students with very poor notation, stating t<t1which means nothing!
(c) When is the particle farthest to the right of the origin? The particle is farthest to the right of the origin at $t = t_3$	1 mark: correct solution given	This was mostly done well.
(d) Sketch the displacement-time graph.	2 marks: correct graph drawn with the features: • start at $x = 10$ • $x = 10$ at t_1 • inflection point at t_2 • maximum turning point at t_3 1 mark: any two of the above clearly shown	Many students did not start their graphs at 10 and failed to maintain a straight line segment until t1. Many students did not associate the maximum turning point with the inflection point and the intercept with the maximum turning point. Overall, this question was done very poorly.

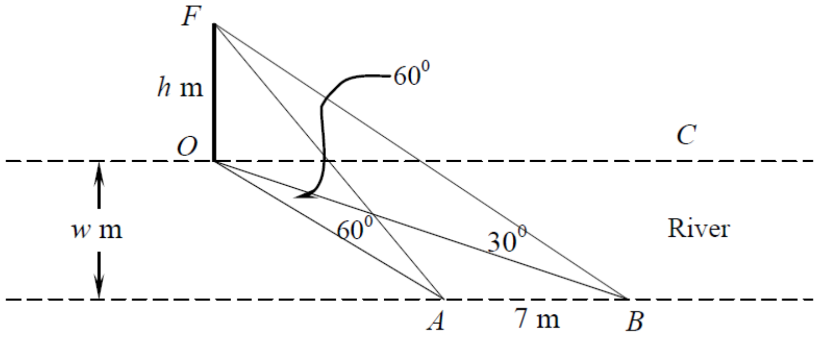


$x(t)$
t
t_1
t_2
t_3
t_4

Question 29 (5 marks)

A river has level parallel riverbanks OC and AB of width w metres. OF is a vertical flagpole of height h metres which stands with its base O on the edge of riverbank OC. Positions A and B are two points on the other riverbank such that $AB = 7$ metres and $\angle AOB = 60°$.

The angle of elevation to the top of the flagpole form A and B are $60°$ and $30°$ respectively, as shown below.



(a) Show that the height of the flagpole is $\sqrt{21}$ metres. In ΔOFB: $\frac{h}{OB} = \tan 30°$ $OB = \frac{h}{\tan 30°}$ $= \sqrt{3}h$ In ΔOFA: $\frac{h}{OA} = \tan 60°$ $OA = \frac{h}{\tan 60°}$ $= \frac{h}{\sqrt{3}}$ In ΔOAB:	3 marks: has found correct expressions for OA and OB in terms of h and used the cosine rule to show that $h = \sqrt{21}$ 2 marks: has found correct expressions for OA and OB in terms of h and used the cosine rule to find h, but has not simplified to show $h = \sqrt{21}$ 1 mark: has found correct expressions for OA and OB in terms of h OR Has attempted to use the cosine rule with incorrect expressions for OA and OB	1 mark: OA = h cot60°, OB = h cot30° Not many got this question correct. Some students mistaken: - ΔOAB as a right-angled triangle. 1 mark is deducted for no working towards h =21, that is not showing the exact values of tan 30°and tan 60°

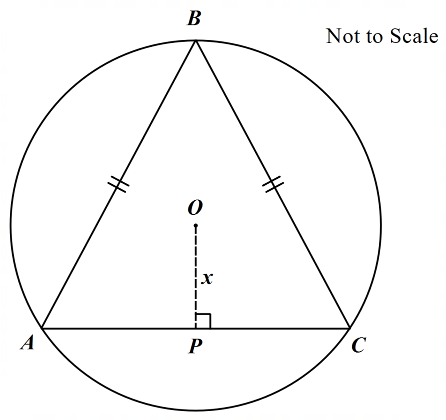
$49 = \frac{10h^2}{3} - h^2$ $49 = \frac{7h^2}{3}$ $h^2 = 21$ $h = \sqrt{21}\ \text{m}, h > 0$		
(b) By finding the area of ΔAOB, or otherwise, find the width of the river. Area of ΔAOB $= \frac{1}{2}(OA)(OB)\sin 60°$ $= \frac{1}{2} \times \sqrt{7} \times 3\sqrt{7} \times \frac{\sqrt{3}}{2}$ $= \frac{21\sqrt{3}}{4}$ $Area\ of\ \Delta AOB = \frac{1}{2} \times w \times 7$ $\frac{7w}{2} = \frac{21\sqrt{3}}{4}$ $7w = \frac{21\sqrt{3}}{2}$ $w = \frac{3\sqrt{3}}{2}$	2 marks: has found the area of the triangle using $A = \frac{1}{2}ab\sin\theta$, and compared that to the area formula $A = \frac{1}{2} base \times width$ to correctly find the width of the river 1 mark: has attempted to use the area formulas in order to find the width, but an error has been made	Most of the students got correct.

Question 30 (4 marks)		
(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ **is** $\frac{6}{9-x^2}$ $\frac{d}{dx}\left[\ln\frac{3+x}{3-x}\right]=\frac{d}{dx}(\ln(3+x))-\frac{d}{dx}(\ln(3-x))$ $=\frac{1}{3+x}-\frac{-1}{3-x}$ $=\frac{(3-x)+(3+x)}{9-x^2}$ $=\frac{6}{9-x^2}$	2 marks: has correctly found the derivative (using log laws or otherwise), and has simplified the expression by putting it over a common denominator 1 mark: has made reasonable progress to find the derivative, using log laws or otherwise	Another method that lot of students used: $\frac{d}{dx}ln\left(f(x)\right)=\frac{f'(x)}{f(x)}$
(b) Hence, or otherwise, find $\int\frac{1}{9-x^2}\,dx$. $\int\frac{1}{9-x^2}dx=\frac{1}{6}\int\frac{6}{9-x^2}dx$ $=\frac{1}{6}\ln\left(\frac{3+x}{3-x}\right)+c$	2 marks: the correct anti-derivative is given with 1/6 and the constant +c 1 mark: the answer is given with the +c, and has omitted the 1/6 coefficient OR The answer has the coefficient 1/6 but has omitted the +c	Quite a few students forgot to include +c.

Question 31 (7 marks)

An isosceles triangle ABC, where $AB = BC$, is inscribed in a circle of radius 10 cm.

$OP = x$ and OP bisects AC, such that $AC \perp OP$.



(a) Show that the area, A, of ΔABC is given by $A = (10+x)\sqrt{100-x^2}$. $BP = 10 + x$ $PC^2 = OC^2 + OP^2$ $= 10^2 - x^2$ $PC = \sqrt{100 - x^2}$ $\therefore AC = 2PC$ $= 2\sqrt{100 - x^2}$ $Area\ of\ \Delta ABC = \frac{1}{2} \times AC \times BP$ $= \frac{1}{2} \times 2\sqrt{100 - x^2} \times (10 + x)$ $= (10 + x)\sqrt{100 - x^2}$	2 marks: has used Pythagoras' theorem to show $PC = \sqrt{100 - x^2}$ and has indicated that the height is $10 + x$ 1 mark: has attempted to use Pythagoras' theorem to show the length of PC, and used the area of a triangle formula	Quite a few students have proved the area formula with the appropriate correct working.

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(b) By first showing that the derivative is

$$\frac{dA}{dx}=\frac{100-10x-2x^2}{\sqrt{100-x^2}}$$

prove that the triangle with maximum area is equilateral.

$$u=10+x,\qquad u'=1$$

$$v=\sqrt{100-x^2},\qquad v'=-x(100-x^2)^{-\frac{1}{2}}$$

$$\frac{dA}{dx}=\sqrt{100-x^2}+(10+x)\times-\frac{x}{\sqrt{100-x^2}}$$

$$=\frac{100-x^2-x(10+x)}{\sqrt{100-x^2}}$$

$$=\frac{100-10x-2x^2}{\sqrt{100-x^2}}$$

$$\text{Solve}\frac{dA}{dx}=0$$

$$100-10x-2x^2=0$$

$$x^2+5x-50=0$$

$$(x+10)(x-5)=0$$

$$x=-10\ or\ x=5$$

Since $x>0$

$$x=5$$

<table>
<tr><td>x</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$\frac{dA}{dx}$</td><td>3.055 ...</td><td>0</td><td>−41.5</td></tr>
<tr><td></td><td>/</td><td>_</td><td>\</td></tr>
</table>

When $x=5$ cm, the triangle has a maximum area.

</td>
<td>

2 marks allocated for the first derivative

2 marks: if applied the quotient rule correctly to find the first derivative

1 mark: finding u’ and v’ and attempt to apply the quotient rule

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3 marks allocated for working towards showing that the triangle with max area is equilateral.

1 mark: for finding a value for x for which there is a max or min

1 mark: for showing that the value for x is a max

1 mark: for showing that the triangle has a maximum area for this value of x and that it is an equilateral triangle.

</td>
<td>

Not done very well.

Not many students got full marks due to omitting answering some part of the question. Example: show the maximum area or the triangle is equilateral.

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</table>

$BP = x + 10$ $= 15$ $PC = \sqrt{100 - 5^2}$ $= 5\sqrt{3}$ $AC = 2\sqrt{100 - 5^2}$ $= 10\sqrt{3}$ $BC = \sqrt{15^2 + \left(5\sqrt{3}\right)^2}$ $= 10\sqrt{3}$ Given $AB = BC$, then $AC = 10\sqrt{3}$ $\therefore$ Since $AB = BC = AC = 10\sqrt{3}$ Then the triangle has a maximum area when it is an equilateral triangle.		