Student Name/Number: ____________________

CARINGBAH HIGH

2020 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- Reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2-4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5-22)

- Attempt Questions 11–40
- Allow about 2 hours and 45 minutes for this section

Marker's Use Only							
Section I	Section II					Total	
Q1-10	Q11-17	Q18-23	Q24-30	Q31-36	Q37-40		
/10	/17	/21	/19	/18	/15	/100	%

Section I

10 marks
Attempt Question 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1) If $\cos\theta = -\frac{12}{13}$ and $180° \le \theta \le 360°$, then $\cot\theta =$

A) $-\frac{5}{12}$ B) $-\frac{12}{5}$

C) $\frac{5}{12}$ D) $\frac{12}{5}$

2) What are the asymptotes of the graph of $y = \frac{1}{x^2 - 9}$

A) $x = \pm\, 3$ B) $x = \pm\, 9$

C) $y = \pm\, 3$ D) $y = \pm\, 9$

3) For the function $f(x) = \frac{x^3}{3} - 5x^2 + 2x + 10$, the gradient is -14 at two points. What are the values of the x-coordinates at these points?

A) $-8,\ 2$ B) $8,\ 2$

C) $8,\ -2$ D) $-8,\ -2$

4) What is the domain of the function $f(x)=\sqrt{x}+\frac{1}{\sqrt{2-x}}$?

A) $(0,\ 2)$

B) $[0,\ 2)$

C) $(0,\ 2]$

D) $[0,\ 2]$

5) If the z scores on an examination are normally distributed and $P(z \le N)=0.6$ for some number N, what is the value of $P(-N \le z \le N)$?

A) 0.1

B) 0.3

C) 0.2

D) 0.4

6) What is the change in amplitude and period when the function $f(x)=\frac{1}{2}\sin 4x$ is transformed into $g(x)=\sin 2x$?

A) The amplitude is halved and the period is halved

B) The amplitude is halved and the period is doubled.

C) The amplitude is doubled and the period is halved

D) The amplitude is doubled and the period is doubled

7) Which statement is true for an ungrouped data set with no outliers?

A) The largest possible range is 2 times the IQR

B) The largest possible range is 3 times the IQR

C) The largest possible range is 4 times the IQR

D) The largest possible range is 5 times the IQR

8) Which line is perpendicular to $3x+4y+7=0$?

A) $4x+3y-7=0$

B) $3x-4y+7=0$

C) $8x-6y-7=0$

D) $4x-7y+7=0$

9) What is the derivative of 3^{4x+5} ?

A) $\ln 3\times 4\times 3^{4x+5}$

B) $(4x+5)\times 3^{4x+5}$

C) $\ln 3\times 3^{4x+5}$

D) $4\times 3^{4x+5}$

10) What is the value of $\ln 2+\ln 4+\ln 8+\ldots+\ln 2^{2n}$?

A) $n^2\ln 2$

B) $n(n+1)\ln 2$

C) $n(n+2)\ln 2$

D) $n(2n+1)\ln 2$

End of Section I

CARINGBAH HIGH

2020 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet

90 marks
Attempt Questions 11–40
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

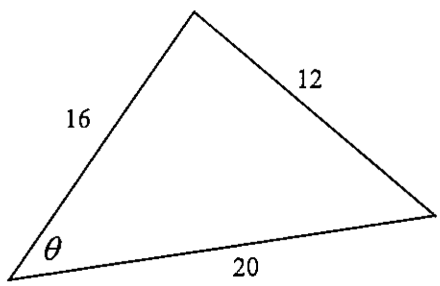
Question 11 (2 marks)

Find the values of a and b (in simplified form) such that

$$\frac{3}{4-\sqrt{2}} = a + \sqrt{b}$$ **2**

Question 12 (2 marks)

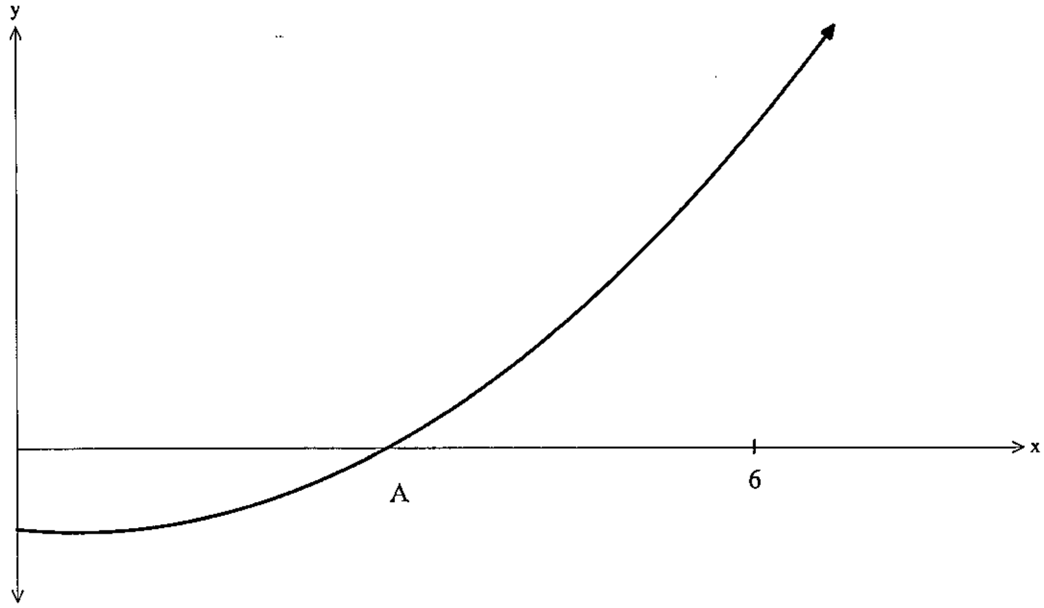
Find the value of θ, correct to the nearest minute **2**



NOT TO SCALE

Question 13 (4 marks)

The diagram below shows the graph of $y = x^2 - x - 6$.



(a) What is the coordinate of A? **1**

(b) Find the area bounded by the x-axis and the curve $y = x^2 - x - 6$ for the interval $0 \leq x \leq 6$. **3**

Question 14 (3 marks)

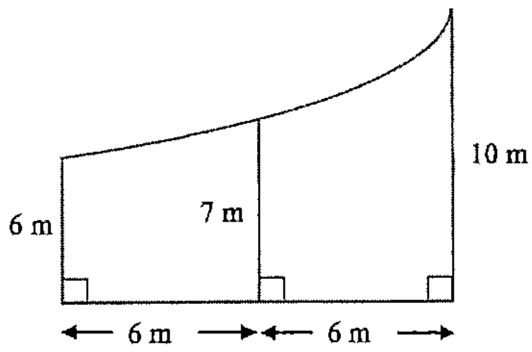
Differentiate

(a) $y = x^2 e^x$ **1**

(b) $f(x) = \dfrac{e^x + 1}{2x}$ **2**

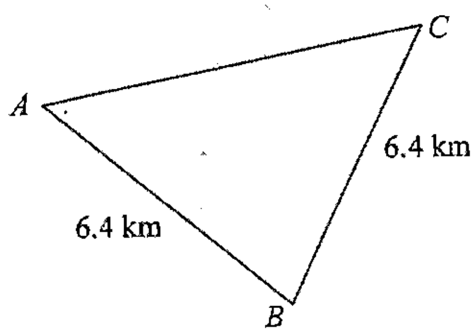
Question 15 (2 marks)

Use two applications of the trapezoidal rule to find an approximation to the area given in the diagram. **2**



NOT TO SCALE

Question 16 (2 marks)



NOT TO SCALE

In the diagram, ABC is a triangular airfield with $AB = BC = 6.4$ km. The bearing of B from A is $140°$ and the bearing of C from B is $032°$.

(a) Show that $\angle ABC = 72°$. **1**

(b) Find the area of the airfield, correct to the nearest square kilometre. **1**

Question 17 (2 marks)

Solve $|2\cos x - 1| = 1$ for $0 \le x \le \pi$ **2**

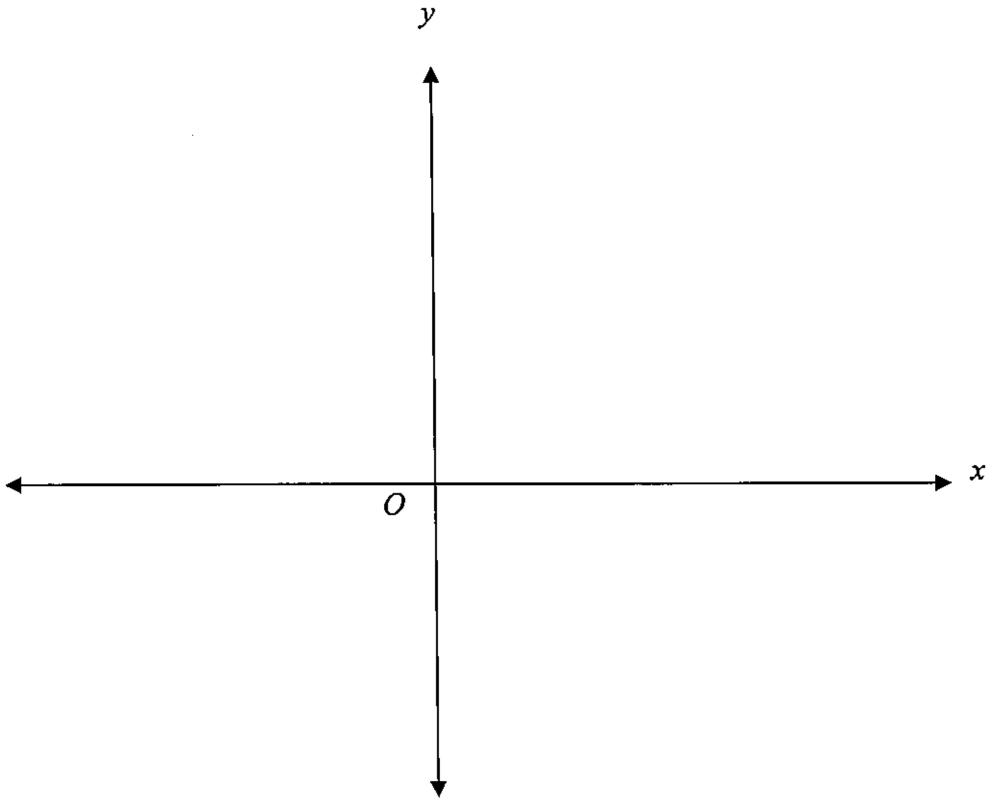
Question 18 (6 marks)

Consider the curve $y = 2x^3 - 9x^2 + 12x$.

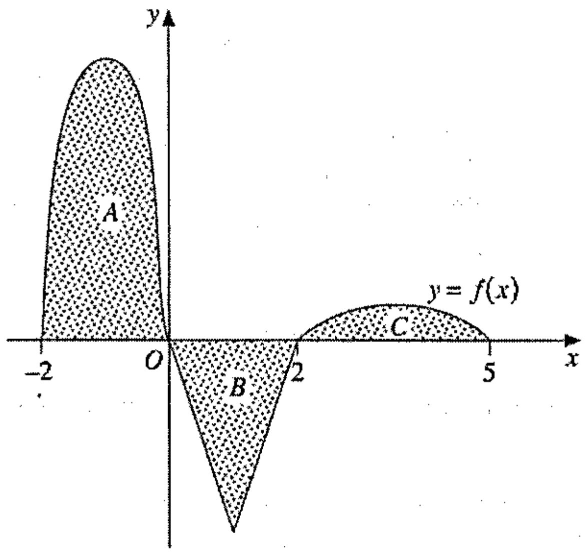
(a) Find the coordinates of the stationary points and determine their nature. **3**

(b) Show that a point of inflection occurs at $x = \frac{3}{2}$. **1**

(c) Sketch the graph $y = 2x^3 - 9x^2 + 12x$, indicating clearly all important features. **2**



Question 19 (1 mark)



NOT TO SCALE

The graph of the function f is shown in the diagram above. The shaded areas are bounded by $y = f(x)$ and the x axis. The shaded area A is 8 square units, the shaded area B is 3 square units and the shaded area C is 1 square unit.

Evaluate $\int_{-2}^{5} f(x)\,dx$.

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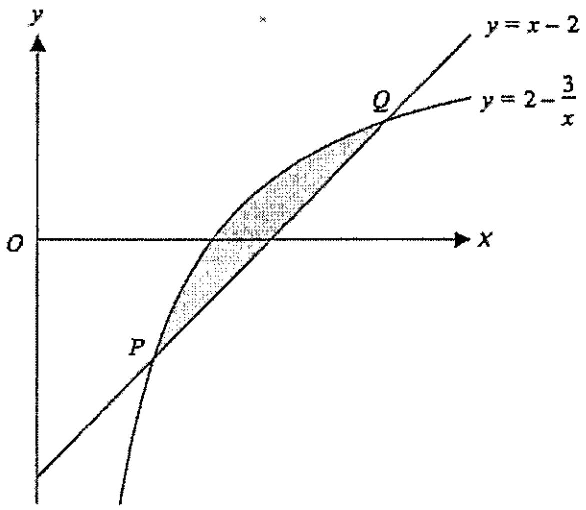
Question 20 (5 marks)



The diagram shows the curves $y = 2 - \dfrac{3}{x}$ and $y = x - 2$ for $x \geq 0$.

(a) Find the coordinates of the two points P and Q where the two curves intersect. **2**

(b) Hence, find in simplest form, the area of the shaded region contained between the two curves. **3**

Question 21 (3 marks)

(a) Show that $\log_x 2 = \dfrac{1}{\log_2 x}$. **1**

(b) Solve the equation $\log_2 x = 4\log_x 2$ **2**

Question 22 (2 marks)

The completion times for the Oztown triathlon race were normally distributed with mean times 60 minutes and standard deviation 5 minutes. Using the empirical rule, find Ozzie's completion time if he finished ahead of 84% of competitors. **2**

Question 23 (4 marks)

The discrete random variable X has a mean of 2 and probability distribution

x	1	2	3	4
$p(x)$	$0 \cdot 3$	0.45	a	b

(a) Show that the two equations in terms of a and b are **2**

$$a + b = 0.25$$
$$3a + 4b = 0.8$$

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(b) Hence find the values of a and b. **2**

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Question 24 (2 marks)

Consider the function $f(x) = e^x$ and $g(x) = \ln(x-2)$.

(a) Find the composite function $f(g(x))$. **1**

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(b) Find the interval notation for the range of the composite function. **1**

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Question 25 (2 marks)

If $y = x\sin 2x$, find $\dfrac{dy}{dx}$ **2**

Question 26 (4 marks)

The table below shows the English marks (x) and the Mathematics marks (y) for a class of 12 students (A-L). Only the English mark is available for student L.

	A	B	C	D	E	F	G	H	I	J	K	L
x	67	61	65	67	75	75	69	85	85	89	87	80
y	58	64	66	68	70	72	72	76	80	82	84	

(a) Calculate the correlation coefficient between x and y for the students A to K. Describe the nature of the correlation coefficient between x and y. **2**

(b) Find the equation of the least squares regression line of y on x for the students A to K. Estimate the Mathematics mark of student L. **2**

Question 27 (2 marks)

If $y = \dfrac{e^x}{x+1}$, find $\dfrac{dy}{dx}$. **2**

Question 28 (2 marks)

Find $\int \tan^2 x \, dx$ **2**

Question 29 (2 marks)

Evaluate $\int_0^2 x(x^2-4)^3 \, dx$ **2**

Question 30 (5 marks)

A metal crate of fixed volume $9\ \text{m}^3$ is to be made in the shape of a rectangular prism with length $2x$ metres, width x metres and height h metres.

(a) Show that the area $A\ \text{m}^2$ of metal required is given by $A = 4x^2 + \dfrac{27}{x}$. **2**

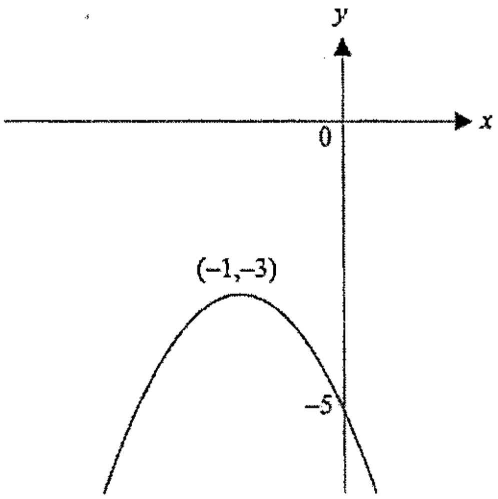
(b) Hence find the minimum area of metal required. **3**

Question 31 (3 marks)

At time (t hours) after 12:00 am, the height (h metres) of the deck of a boat above the level of the jetty is given by $h = 2\cos\left(\frac{4\pi}{25}t\right) + 1$. Find, correct to the nearest minute, the first time after 12:00 am when the deck of the boat is level with the jetty. **3**

Question 32 (3 marks)

The function $f(x) = x^2$ is transformed into a new function whose graph is shown in the diagram below.



NOT TO SCALE

Find the equation of the new function in the form $g(x) = k\,f(x+b)+c$ for some constants k, b and c. **3**

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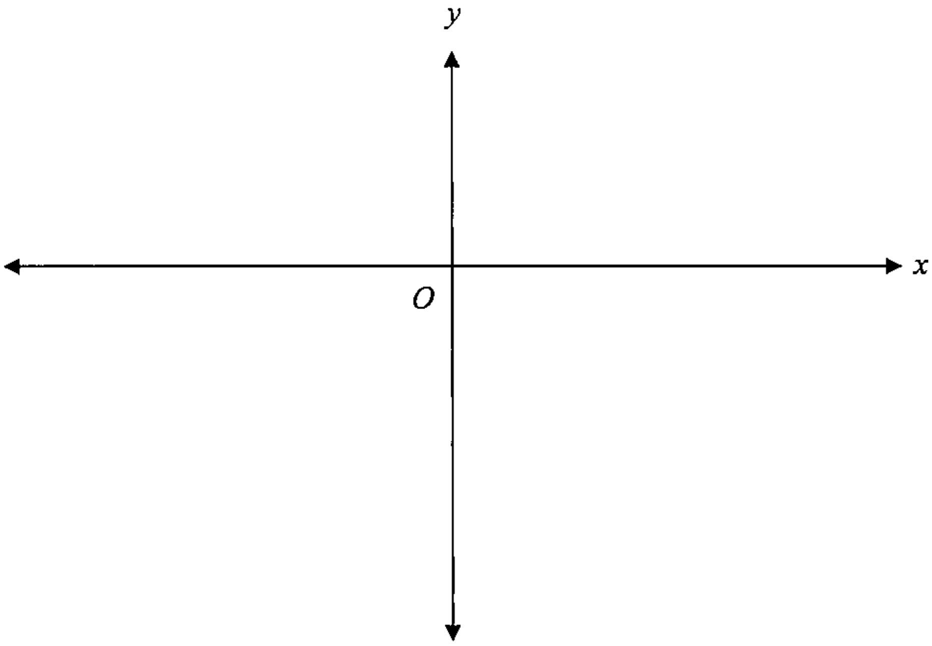
Question 33 (3 marks)

(a) On the number plane below, draw the graphs of $y = \cos \pi x$ and $y = 1 - |x|$ for $-3 \le x \le 3$. **2**



(b) Hence find the number of solutions of the equation $\cos \pi x = 1 - |x|$ in the domain $(-\infty, \infty)$. **1**

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Question 34 (3 marks)

If $y = \tan^2 x$, find the values of the constants a and b, such that $\dfrac{d^2y}{dx^2} = ay^2 + by + 2$. **3**

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Question 35 (3 marks)

The continuous random variable X has probability density function $f(x)=\frac{1}{2}\sin x$ for $0 \le x \le \pi$.

(a) Find the cumulative distributive function (CDF) **2**

(b) Find the first quartile of the distribution. **1**

Question 36 (3 marks)

(a) Differentiate $x\log_e x$. **1**

(b) Hence or otherwise, evaluate (in exact form), $\int_1^2 \log_e x \, dx$. **2**

Question 37 (4 marks)

At time t years after it was purchased the value $\$V$ of a car is given by $V = 25\ 000e^{-0.5t}$.

(a) Find the loss in value of the car during the third year. **1**

(b) Find the year in which the car is losing value at a rate of \$100 per year. **2**

Question 38 (2 marks)

The first term of a geometric series is 16 and the fourth term is $\frac{1}{4}$.

(a) Find the common ratio. **1**

(b) Find the limiting sum of the series. **1**

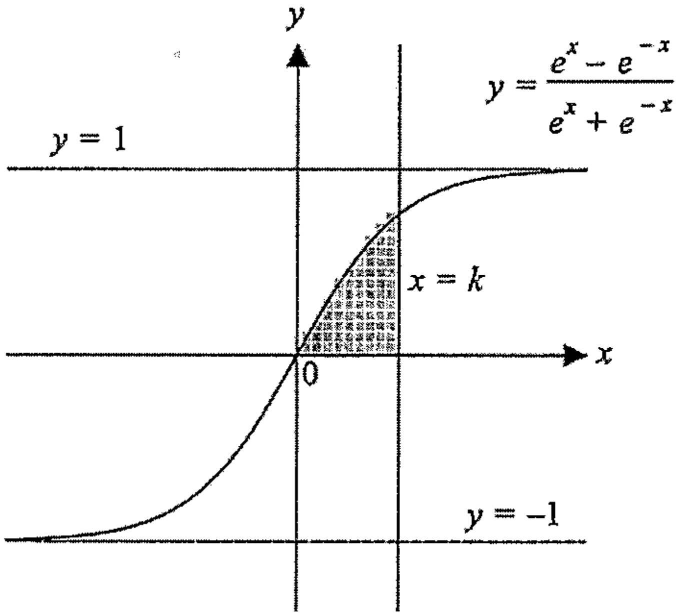
Question 39 (5 marks)

A particle is moving in a straight line. At time t seconds it has a displacement x metres from a fixed point O on the line, velocity v ms^{-1}, and acceleration a ms^{-2} is given by $a = 6t - 12$. Initially, the particle is at rest at O.

(a) Find expressions for v and x in terms of t. **3**

(b) Find when and where the particle is next at rest. **2**

Question 40 (5 marks)



NOT TO SCALE

The diagram shows the graph of the curve $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(a) Show that the shaded region bounded by the curve, the x axis and the line

$x = k$, where $k > 0$, has area $\ln\left(\dfrac{e^k + e^{-k}}{2}\right)$. **2**

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Student Name/Number: solutions

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Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
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Total marks: 100

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Marker's Use Only							
Section I	**Section II**					**Total**	
Q1-10	**Q11-17**	**Q18-23**	**Q24-30**	**Q31-36**	**Q37-40**		
/10	/17	/21	/19	/18	/15	/100	%

Section I

10 marks
Attempt Question 1-10
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Use the multiple-choice answer sheet for Questions 1-10.

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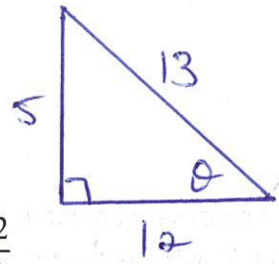
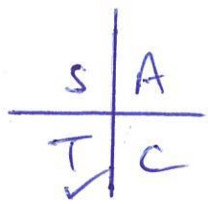
1) If $\cos\theta = -\dfrac{12}{13}$ and $180° \le \theta \le 360°$, then $\cot\theta =$





A) $-\dfrac{5}{12}$ B) $-\dfrac{12}{5}$

C) $\dfrac{5}{12}$ D) $\dfrac{12}{5}$

$\cot\theta = \dfrac{1}{\tan\theta}$
$= \dfrac{1}{5/12}$
$= \dfrac{12}{5}$

2) What are the asymptotes of the graph of $y = \dfrac{1}{x^2 - 9}$

A) $x = \pm\, 3$ B) $x = \pm\, 9$

C) $y = \pm\, 3$ D) $y = \pm\, 9$

$x^2 - 9 = 0$
$(x-3)(x+3) = 0$
$x = \pm 3$

3) For the function $f(x) = \dfrac{x^3}{3} - 5x^2 + 2x + 10$, the gradient is -14 at two points. What are the values of the x-coordinates at these points?

$f'(x) = x^2 - 10x + 2$
$x^2 - 10x + 2 = -14$
$x^2 - 10x + 16 = 0.$
$(x-8)(x-2) = 0.$
$x = 8, 2$

A) $-8,\ 2$ B) $8,\ 2$

C) $8,\ -2$ D) $-8,\ -2$

4) What is the domain of the function $f(x)=\sqrt{x}+\frac{1}{\sqrt{2-x}}$?

$x \geq 0,\ x < 2$
$0 \leq x < 2$
$= [0, 2)$

A) $(0,\ 2)$ B) $[0,\ 2)$

C) $(0,\ 2]$ D) $[0,\ 2]$

5) If the z scores on an examination are normally distributed and $P(z \leq N) = 0.6$ for some number N, what is the value of $P(-N \leq z \leq N)$?

$P(-N \leq Z \leq N)$
$= 2P(0 \leq Z \leq N)$
$= 2\{P(Z \leq N) - 0.5\}$
$= 2(0.6 - 0.5)$
$= 2(0.1)$
$= 0.2$

A) 0.1 B) 0.3

C) 0.2 D) 0.4

6) What is the change in amplitude and period when the function $f(x)=\frac{1}{2}\sin 4x$ is transformed into $g(x)=\sin 2x$?

amplitude $= \frac{1}{2}$, period $= \frac{\pi}{2}$
amplitude $= 1$, period $= \pi$

A) The amplitude is halved and the period is halved

B) The amplitude is halved and the period is doubled.

C) The amplitude is doubled and the period is halved

D) The amplitude is doubled and the period is doubled

7) Which statement is true for an ungrouped data set with no outliers?

A) The largest possible range is 2 times the IQR

B) The largest possible range is 3 times the IQR

C) The largest possible range is 4 times the IQR

D) The largest possible range is 5 times the IQR

$(1.5+1+1.5)\times IQR$
$= 4\times IQR$

8) Which line is perpendicular to $3x+4y+7=0$?

$4y=-3x-7$

$y=-\frac{3}{4}x-\frac{7}{4}$

$\therefore m=-\frac{3}{4}$

$m_{\perp}=\frac{4}{3}.$

A) $4x+3y-7=0$ B) $3x-4y+7=0$

(C)) $8x-6y-7=0$ D) $4x-7y+7=0$

$6y=8x-7$

$y=\frac{8x}{6}-\frac{7}{6}$

$y=\frac{4}{3}x-\frac{7}{6}$

9) What is the derivative of 3^{4x+5} ?

(A)) $\ln 3\times 4\times 3^{4x+5}$ B) $(4x+5)\times 3^{4x+5}$

$\frac{d}{dx}3^{4x+5}$

$=\ln 3\times 4\times 3^{4x+5}$

C) $\ln 3\times 3^{4x+5}$ D) $4\times 3^{4x+5}$

10) What is the value of $\ln 2+\ln 4+\ln 8+...+\ln 2^{2n}$?

A) $n^2\ln 2$ B) $n(n+1)\ln 2$

C) $n(n+2)\ln 2$ (D)) $n(2n+1)\ln 2$

$\ln 2+\ln 2^2+\ln 2^3+...+\ln 2^{2n}$

$=\ln 2+2\ln 2+3\ln 2+...+2n\ln 2$

$=(1+2+3+...+2n)\ln 2$

$a=1, d=1, n=2n, l=2n$

End of Section I

$S_n=\frac{n}{2}(a+l)$

$=\frac{2n}{2}(1+2n)$

$=n(1+2n)$

$\therefore$ sum $=n(1+2n)\ln 2$

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- Your responses should include relevant mathematical reasoning and/or calculations.
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Question 11 (2 marks)

Find the values of a and b (in simplified form) such that

$$\frac{3}{4-\sqrt{2}} = a + \sqrt{b}$$ **2**

$\frac{3}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$

$= \frac{12+3\sqrt{2}}{16-4}$

$= \frac{12+3\sqrt{2}}{14}$

$= \frac{6}{7} + \frac{3}{14}\sqrt{2}$

$\therefore a = \frac{6}{7}, \sqrt{b} = \frac{3}{14}\sqrt{2}$

$b = \left(\frac{3}{14}\sqrt{2}\right)^2$

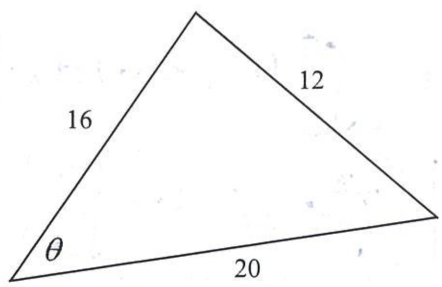
$= \frac{9 \times 2}{196}$

$= \frac{18}{196}$

$\therefore b = \frac{9}{98}$.

Question 12 (2 marks)

Find the value of θ, correct to the nearest minute **2**



NOT TO SCALE

$\cos\theta = \frac{16^2 + 20^2 - 12^2}{2 \times 16 \times 20}$

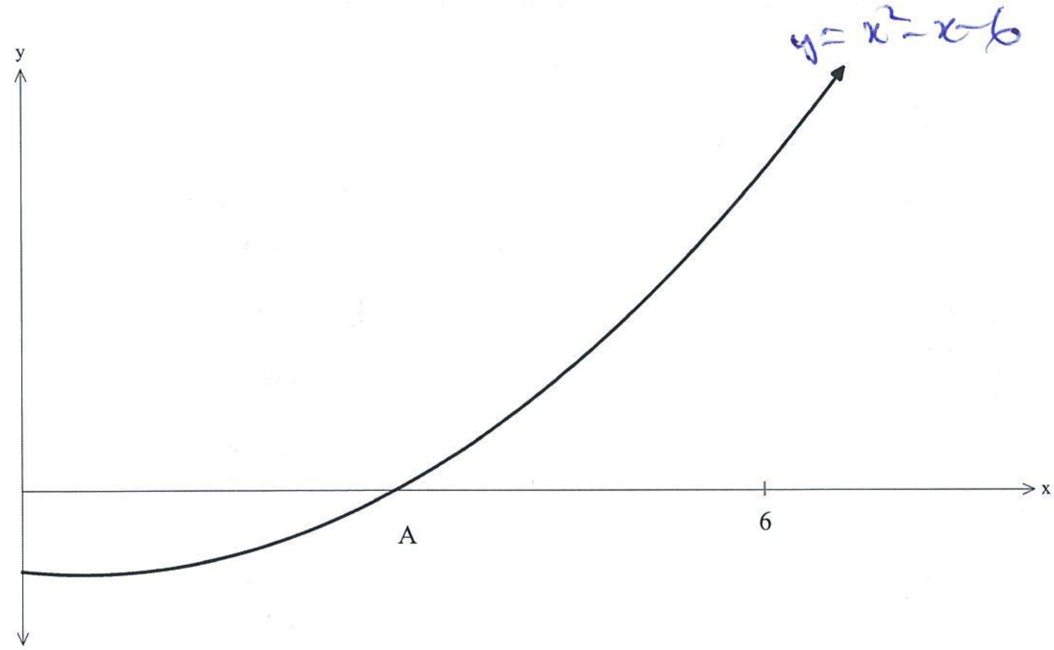
$= \frac{4}{5}$.

$\theta = \cos^{-1}\left(\frac{4}{5}\right)$

$= 36°52'$

Question 13 (4 marks)

The diagram below shows the graph of $y = x^2 - x - 6$.



(a) What is the coordinate of A? **1**

$x^2 - x - 6 = 0.$

$(x-3)(x+2) = 0.$

$\therefore x = 3, -2$

$\therefore A(3, 0)$

(b) Find the area bounded by the x-axis and the curve $y = x^2 - x - 6$ for the interval $0 \le x \le 6$. **3**

$A = \int_3^6 (x^2 - x - 6)\,dx + \left| \int_0^3 (x^2 - x - 6)\,dx \right|$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_3^6 + \left| \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_0^3 \right|$

$= \left(\frac{6^3}{3} - \frac{6^2}{2} - 6(6)\right) - \left(\frac{3^3}{3} - \frac{3^2}{2} - 6(3)\right) + \left| \left(\frac{3^3}{3} - \frac{3^2}{2} - 6(3)\right) - 0 \right|$

$= 18 + \frac{27}{2} + \frac{27}{2}$

$= 45u^2$

Question 14 (~~4~~ 3 marks)

Differentiate

(a) $y = x^2 e^x$ **1**

$y' = x^2 e^x + 2xe^x$

$= xe^x(x+2)$

(b) $f(x) = \dfrac{e^x + 1}{2x}$ **2**

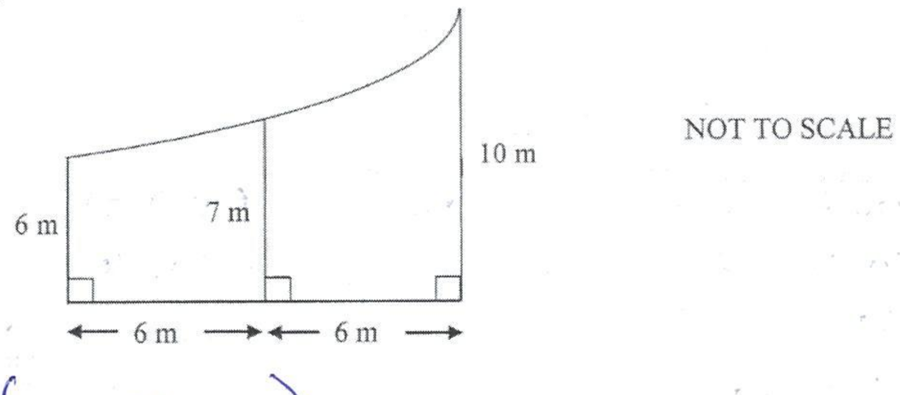
$f'(x) = \dfrac{2x(e^x) - (e^x+1)2}{4x^2}$

$= \dfrac{2xe^x - 2e^x - 2}{4x^2}$

$= \dfrac{xe^x - e^x - 1}{2x^2}$

Question 15 (2 marks)

Use two applications of the trapezoidal rule to find an approximation to the area given in the diagram. **2**

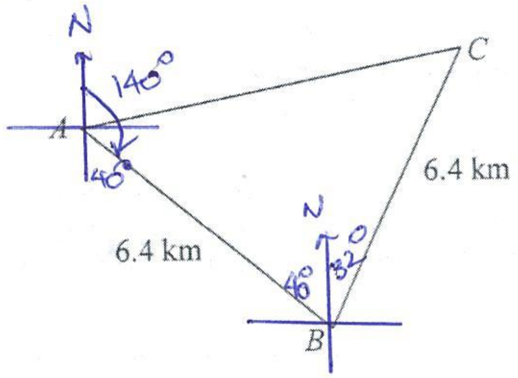


$A \doteq \frac{6}{2}(6 + 2(7) + 10)$

$\doteq 3(30)$

$\doteq 90\,m^2$

Question 16 (2 marks)



NOT TO SCALE

In the diagram, ABC is a triangular airfield with $AB = BC = 6.4$ km . The bearing of B from A is 140° and the bearing of C from B is 032°.

(a) Show that $\angle ABC = 72°$. **1**

$\angle ABC = \angle ABN + \angle NBC$

$= 40° + 32°$ (alt. ∠s =, ∥ lines)

$= 72°$

(b) Find the area of the airfield, correct to the nearest square kilometre. **1**

$A = \frac{1}{2} \times 6.4 \times 6.4 \sin 72°$

$= 19 \text{ km}^2$

Question 17 (2 marks)

Solve $|2\cos x - 1| = 1$ for $0 \le x \le \pi$ **2**

$2\cos x - 1 = 1$ or $2\cos x - 1 = -1$

$2\cos x = 2$ $2\cos x = 0$

$\cos x = 1$ $\cos x = 0$

$\therefore x = 0$ $x = \frac{\pi}{2}$

$\therefore x = 0, \frac{\pi}{2}$

Question 18 (6 marks)

Consider the curve $y = 2x^3 - 9x^2 + 12x$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

$y' = 6x^2 - 18x + 12$

$y' = 0$, stat. pt

$6x^2 - 18x + 12 = 0$.

$x^2 - 3x + 2 = 0$.

$(x-2)(x-1) = 0$.

$x = 1, 2$.

sub $x = 1$, $y = 5$

sub $x = 2$, $y = 4$.

$\therefore (1,5)$ & $(2,4)$

$y'' = 12x - 8$

sub $x = 1$, $y'' = -6$

< 0, max

sub $x = 2$, $y'' = 6$

> 0, min.

$\therefore (1,5)$ is a max & $(2,4)$ is a min stationary point.

(b) Show that a point of inflection occurs at $x = \frac{3}{2}$. **1**

$y'' = 0$, $12x - 8 = 0$.

$12x = 8$

$x = 8/12$

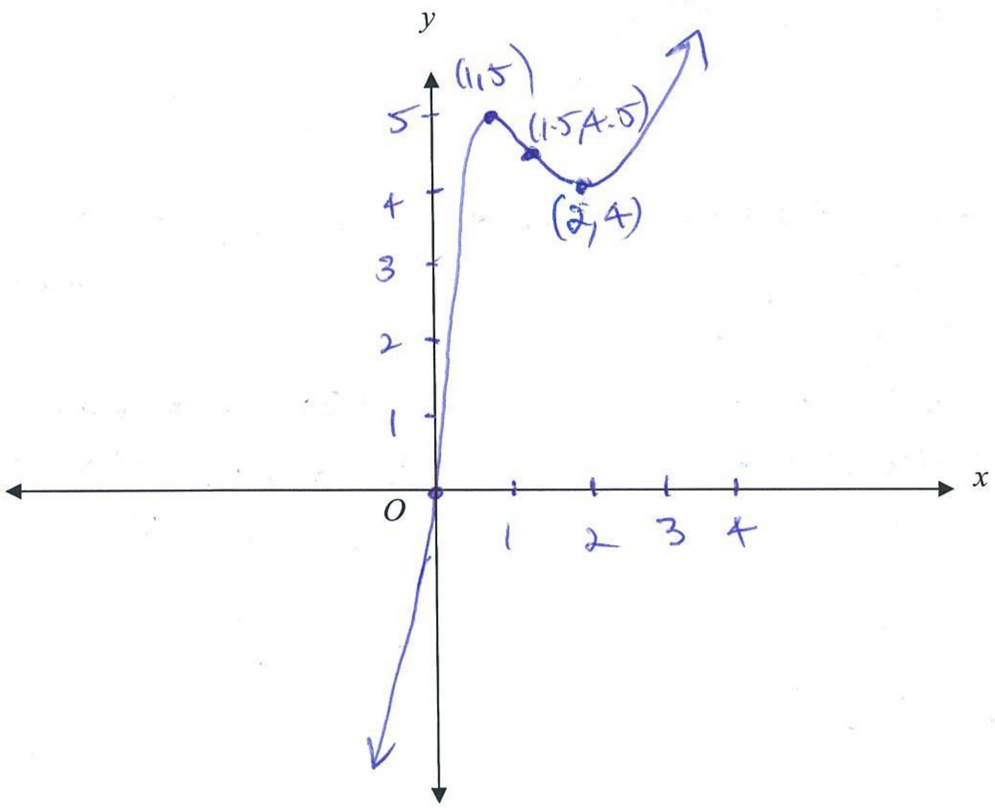
$x = \frac{3}{2}$

x	1.25	1.5	1.75
y''	-3	0	3

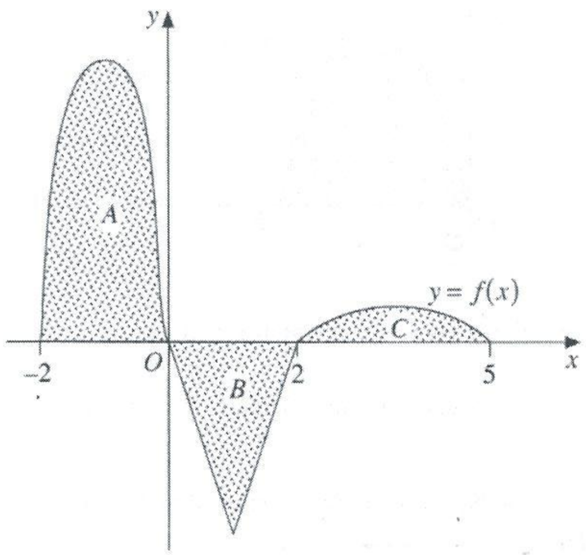
$\therefore$ change in concavity.

$\therefore$ inflection point occurs at $x = \frac{3}{2}$

(c) Sketch the graph $y = 2x^3 - 9x^2 + 12x$, indicating clearly all important features. **2**



Question 19 (1 mark)



NOT TO SCALE

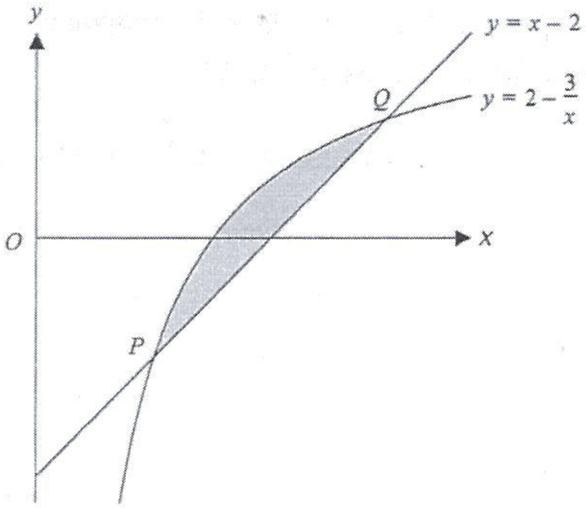
The graph of the function *f* is shown in the diagram above. The shaded areas are bounded by $y = f(x)$ and the *x* axis. The shaded area *A* is 8 square units, the shaded area *B* is 3 square units and the shaded area *C* is 1 square unit.

Evaluate $\int_{-2}^{5} f(x)\, dx$.

$\int_{-2}^{5} f(x) = 8 - 3 + 1$

$= 6u^2$

Question 20 (5 marks)



NOT TO SCALE

The diagram shows the curves $y = 2 - \frac{3}{x}$ and $y = x - 2$ for $x \geq 0$.

(a) Find the coordinates of the two points P and Q where the two curves intersect. **2**

$2 - \frac{3}{x} = x - 2$

$2x - 3 = x^2 - 2x$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$

when $x = 3$, $y = 1$

when $x = 1$, $y = -1$

$\therefore P(1, -1)$ & $Q(3, 1)$

(b) Hence, find in simplest form, the area of the shaded region contained between the two curves. **3**

$A = \int_1^3 \left[2 - \frac{3}{x} - (x-2)\right] dx$

$= \int_1^3 \left(2 - \frac{3}{x} - x + 2\right) dx$

$= \int_1^3 \left(4 - \frac{3}{x} - x\right) dx$

$= \left[4x - 3\ln x - \frac{x^2}{2}\right]_1^3$

$= \left(4(3) - 3\ln 3 - \frac{3^2}{2}\right) - \left(4 - 3\ln 1 - \frac{1}{2}\right)$

$= 12 - 3\ln 3 - \frac{9}{2} - 4 + \frac{1}{2}$

$= (4 - 3\ln 3)\ u^2$

Question 21 (3 marks)

(a) Show that $\log_x 2 = \dfrac{1}{\log_2 x}$. **1**

$$\log_x 2 = \frac{\log_2 2}{\log_2 x} \quad \text{(change of base rule)}$$

$$= \frac{1}{\log_2 x}$$

(b) Solve the equation $\log_2 x = 4\log_x 2$ **2**

$$\log_2 x = 4 \times \frac{1}{\log_2 x}.$$

$$\log_2 x = \frac{4}{\log_2 x}$$

$$(\log_2 x)^2 = 4$$

$$\log_2 x = \pm 2$$

$\therefore \log_2 x = 2$ & $\log_2 x = -2$

$x = 2^2$ $\quad$ $x = 2^{-2}$

$x = 4$ $\quad$ $x = \frac{1}{4}$

Question 22 (2 marks)

The completion times for the Oztown triathlon race were normally distributed with mean times 60 minutes and standard deviation 5 minutes. Using the empirical rule, find Ozzie's completion time if he finished ahead of 84% of competitors. **2**

$50\% + \frac{1}{2} \times 68\% = 84\%$ (normal distribution)

84% of the times are $> \mu - \sigma$

$60 - 5 = 55$ minutes

Question 23 (4 marks)

The discrete random variable X has a mean of 2 and probability distribution

x	1	2	3	4
$p(x)$	$0\cdot3$	0.45	a	b

(a) Show that the two equations in terms of a and b are **2**

$$a+b=0.25$$
$$3a+4b=0.8$$

$0\cdot3+0\cdot45+a+b=1$

$0\cdot75+a+b=1$

$a+b=0\cdot25$

$0\cdot3+2(0\cdot45)+3a+4b=2$

$0\cdot3+0\cdot9+3a+4b=2$

$1\cdot2+3a+4b=2$

$3a+4b=0\cdot8$

(b) Hence find the values of a and b. **2**

$4a+4b=1 \ldots (1)$

$-\ 3a+4b=0\cdot8 \ldots (2)$

(1)-(2) $a=0\cdot2$

sub $a=0\cdot2$ into (1)

$4(0\cdot2)+4b=1$

$0\cdot8+4b=1$

$4b=0\cdot2$

$b=0\cdot05$

$\therefore a=0\cdot2,\ b=0\cdot05$

Question 24 (2 marks)

Consider the function $f(x)=e^x$ and $g(x)=\ln(x-2)$.

(a) Find the composite function $f(g(x))$. **1**

$f(g(x))=f(\ln(x-2))$

$=e^{\ln(x-2)}$

$=x-2,\ x>2.$

(b) Find the interval notation for the range of the composite function. **1**

$(0,+\infty)$

Question 25 (2 marks)

If $y = x\sin 2x$, find $\dfrac{dy}{dx}$ **2**

$y' = x(2\cos 2x) + \sin 2x(1)$

$= 2x\cos 2x + \sin 2x$

Question 26 (4 marks)

The table below shows the English marks (x) and the Mathematics marks (y) for a class of 12 students (A-L). Only the English mark is available for student L.

	A	B	C	D	E	F	G	H	I	J	K	L
x	67	61	65	67	75	75	69	85	85	89	87	80
y	58	64	66	68	70	72	72	76	80	82	84	

(a) Calculate the correlation coefficient between x and y for the students A to K. Describe the nature of the correlation coefficient between x and y. **2**

$r = 0.9$, strong positive correlation

(b) Find the equation of the least squares regression line of y on x for the students A to K. Estimate the Mathematics mark of student L. **2**

$y = 18 + 0.72x$.

when $x = 80$, $y = 18 + 0.72(80)$

$y = 75.6$

$\therefore L \doteqdot 76$.

Question 27 (2 marks)

If $y = \dfrac{e^x}{x+1}$, find $\dfrac{dy}{dx}$. **2**

$$\frac{dy}{dx} = \frac{(x+1)e^x - e^x(1)}{(x+1)^2}$$

$$= \frac{xe^x + e^x - e^x}{(x+1)^2}$$

$$= \frac{xe^x}{(x+1)^2}$$

Question 28 (2 marks)

Find $\int \tan^2 x \, dx$ **2**

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$

$$= \tan x - x + c$$

Question 29 (2 marks)

Evaluate $\int_0^2 x(x^2-4)^3 \, dx$ **2**

$$\frac{1}{2}\int_0^2 2x(x^2-4)^3 \, dx$$

$$= \frac{1}{2}\left[\frac{(x^2-4)^4}{4}\right]_0^2$$

$$= \frac{1}{8}\left[(x^2-4)^4\right]_0^2$$

$$= \frac{1}{8}\left[(2^2-4)^4 - (0-4)^4\right]$$

$$= \frac{1}{8}(0-256)$$

$$= -32$$

Question 30 (5 marks)

A metal crate of fixed volume 9 m^3 is to be made in the shape of a rectangular prism with length $2x$ metres, width x metres and height h metres.

(a) Show that the area $A\text{ m}^2$ of metal required is given by $A = 4x^2 + \dfrac{27}{x}$. **2**

$V = Ah$

$9 = 2x^2 h$

$h = \dfrac{9}{2x^2}$ ... ①

$A = 2(2x^2) + 2(2xh) + 2(xh)$

$= 4x^2 + 4xh + 2xh$

$= 4x^2 + 6xh$ ... ②

sub ① into ②.

$A = 4x^2 + 6x\left(\dfrac{9}{2x^2}\right)$

$= 4x^2 + \dfrac{27}{x}$

(b) Hence find the minimum area of metal required. **3**

$A = 4x^2 + 27x^{-1}$

$\dfrac{dA}{dx} = 8x - 27x^{-2}$

$= 8x - \dfrac{27}{x^2}$

$\dfrac{dA}{dx} = 0, \quad 8x - \dfrac{27}{x^2} = 0$

$8x^3 - 27 = 0$

$8x^3 = 27$

$x^3 = \dfrac{27}{8}$

$x = \dfrac{3}{2}$

$\dfrac{d^2A}{dx^2} = 8 + 54x^{-3}$

$= 8 + \dfrac{54}{x^3}$

sub $x = \dfrac{3}{2}$, $\dfrac{d^2A}{dx^2} = 8 + \dfrac{54}{(3/2)^3}$

$= 24$

> 0, min

$\therefore A_{min} = 4\left(\dfrac{3}{2}\right)^2 + \dfrac{27}{(3/2)}$

$= 27\text{ m}^2$

Question 31 (3 marks)

At time (t hours) after 12:00 am, the height (h metres) of the deck of a boat above the level of the jetty is given by $h = 2\cos\left(\frac{4\pi}{25}t\right) + 1$. Find, correct to the nearest minute, the first time after 12:00 am when the deck of the boat is level with the jetty. **3**

$h = 2\cos\left(\frac{4\pi}{25}t\right) + 1$

when $h = 0$,

$2\cos\left(\frac{4\pi}{25}t\right) + 1 = 0$

$2\cos\left(\frac{4\pi}{25}t\right) = -1$

$\cos\left(\frac{4\pi}{25}t\right) = -\frac{1}{2}$

$\frac{4\pi}{25}t = \cos^{-1}\left(-\frac{1}{2}\right)$

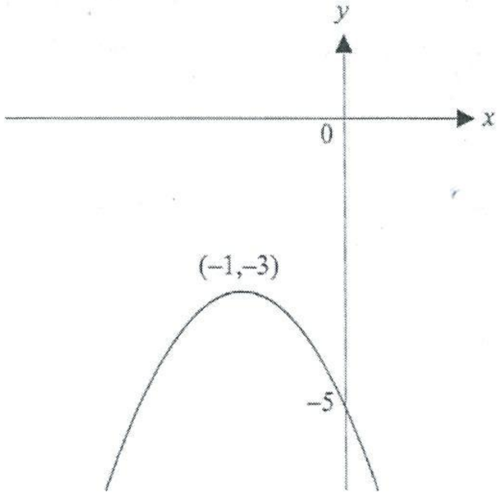
$\frac{4\pi}{25}t = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$

$t = \frac{25}{6}, \frac{25}{3}, \ldots$

$= 4{:}10\text{am}$

Question 32 (3 marks)

The function $f(x)=x^2$ is transformed into a new function whose graph is shown in the diagram below.



NOT TO SCALE

Find the equation of the new function in the form $g(x)=k\,f(x+b)+c$ for some constants k, b and c. **3**

The curve is reflected in the x-axis, dilated vertically then translated 1 unit to the left and down by 3 units.

$\therefore g(x)=k(x+1)-3$

$g(0)=-5$

$\therefore k(0+1)-3=-5$

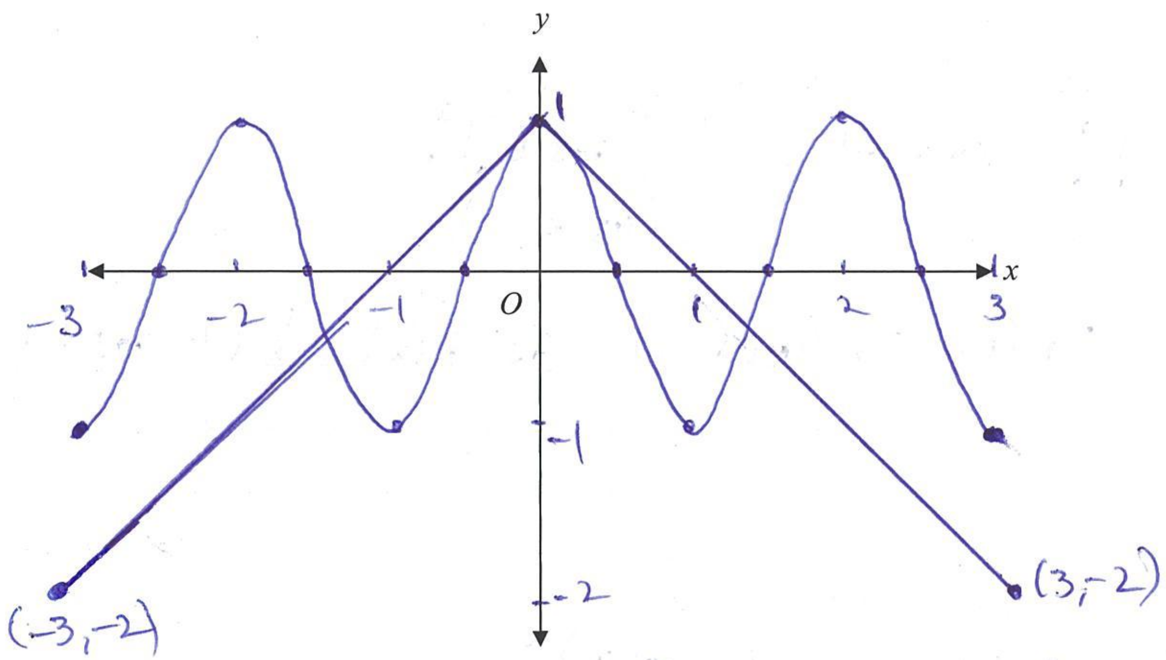
$k-3=-5$

$k=-2$

$\therefore g(x)=-2(x+1)^2-3$

Question 33 (3 marks)

(a) On the number plane below, draw the graphs of $y = \cos \pi x$ and $y = 1 - |x|$ for $-3 \le x \le 3$. **2**



(b) Hence find the number of solutions of the equation $\cos \pi x = 1 - |x|$ in the domain $(-\infty, \infty)$. **1**

5 times.

Question 34 (3 marks)

If $y = \tan^2 x$, find the values of the constants a and b, such that $\frac{d^2y}{dx^2} = ay^2 + by + 2$. **3**

$\frac{dy}{dx} = 2\tan x \sec^2 x$

$= 2\tan x(1 + \tan^2 x)$

$= 2\tan x + 2\tan^3 x$

$\frac{d^2y}{dx^2} = 2\sec^2 x + 6\tan^2 x \sec^2 x$

$= 2(1 + \tan^2 x) + 6\tan^2 x(1 + \tan^2 x)$

$= 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x$

$= 2 + 8\tan^2 x + 6\tan^4 x$

sub $y = \tan^2 x$,

$\frac{d^2y}{dx^2} = 2 + 8y + 6y^2$

$\therefore a = 6, \ b = 8$

Question 35 (3 marks)

The continuous random variable X has probability density function $f(x) = \frac{1}{2}\sin x$ for $0 \le x \le \pi$.

(a) Find the cumulative distributive function (CDF) **2**

$\int_0^x \frac{1}{2}\sin t\, dt = -\frac{1}{2}\left[\cos t\right]_0^x$

$= -\frac{1}{2}(\cos x - 1)$

$\therefore F(x) = -\frac{1}{2}(\cos x - 1)$ or $F(x) = \frac{1}{2}(1 - \cos x)$

(b) Find the first quartile of the distribution. **1**

$F(x) = 0.25$, $\frac{1}{2}(1 - \cos x) = \frac{1}{4}$

$1 - \cos x = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

Question 36 (3 marks)

(a) Differentiate $x\log_e x$. **1**

$\frac{d}{dx}(x\log_e x) = x\left(\frac{1}{x}\right) + \log_e x(1)$

$= 1 + \log_e x$

$\therefore \log_e x = \frac{d}{dx}(x\log_e x) - 1$

(b) Hence or otherwise, evaluate (in exact form), $\int_1^2 \log_e x\, dx$. **2**

$\int_1^2 \log_e x\, dx = \left[x\log_e x - x\right]_1^2$

$= 2\log_e 2 - 2 - (\log_e 1 - 1)$

$= 2\log_e 2 - 2 + 1$

$= 2\log_e 2 - 1$

Question 37 (~~4~~ 3 marks)

At time t years after it was purchased the value $\$V$ of a car is given by $V = 25\ 000e^{-0.5t}$.

(a) Find the loss in value of the car during the third year. **1**

$V = 25000e^{-0.5t}$

$\frac{dV}{dt} = -12500e^{-0.5t}$

(b) Find the year in which the car is losing value at a rate of \$100 per year. **2**

$-12500e^{-0.5t} = -100$

$12500e^{-0.5t} = 100$

$e^{-0.5t} = \frac{1}{125}$

$\ln e^{-0.5t} = \ln\left(\frac{1}{125}\right)$

$-0.5t = \ln 125^{-1}$

$-0.5t = -\ln 125$

$0.5t = \frac{\ln 125}{0.5}$

$t \doteqdot 9.6566$

$\therefore$ during the 10th yr.

Question 38 (2 marks)

The first term of a geometric series is 16 and the fourth term is $\frac{1}{4}$.

(a) Find the common ratio. $a = 16,\ T_4 = \frac{1}{4}$ **1**

$T_n = ar^{n-1}$

$\frac{1}{4} = 16r^{4-1}$

$\frac{1}{4} = 16r^3$

$r^3 = \frac{1}{64}$

$r = \frac{1}{4}$

(b) Find the limiting sum of the series. **1**

$S_\infty = \frac{a}{1-r}$

$= \frac{16}{1-1/4}$

$= 21\frac{1}{3}$

Question 39 (5 marks)

A particle is moving in a straight line. At time t seconds it has a displacement x metres from a fixed point O on the line, velocity v ms^{-1}, and acceleration a ms^{-2} is given by $a = 6t - 12$. Initially, the particle is at rest at O.

(a) Find expressions for v and x in terms of t. **3**

$a = 6t - 12$

$v = \frac{6t^2}{2} - 12t + c$

$v = 3t^2 - 12t + c$

when $t=0$, $v=0$ $\therefore c=0$.

$v = 3t^2 - 12t$

$x = \frac{3t^3}{3} - \frac{12t^2}{2} + c$

$x = t^3 - 6t^2 + c$

when $t=0$, $x=0$ $\therefore c=0$

$\therefore x = t^3 - 6t^2$

(b) Find when and where the particle is next at rest. **2**

At rest, $v=0$.

$3t^2 - 12t = 0$.

$3t(t-4) = 0$.

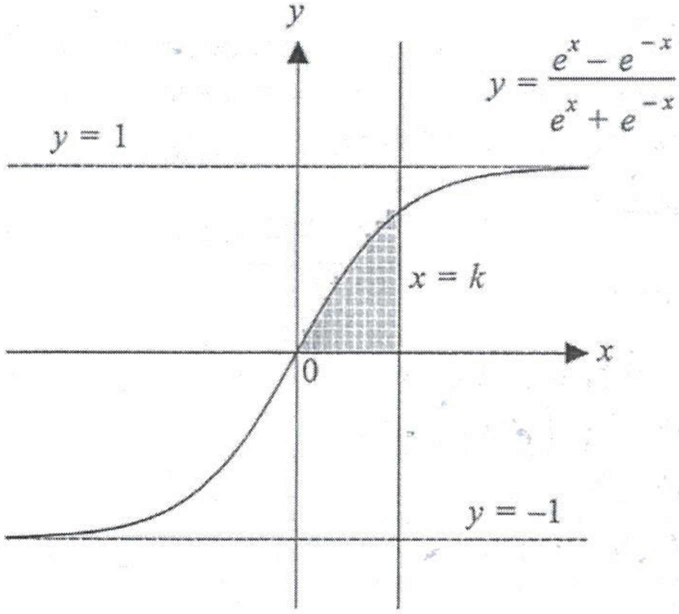
$t = 0, 4$

when $t=4$, $x = 4^3 - 6(4)^2$

$= -32$.

$\therefore$ particle next at rest is when $t=4$ sec. when it is 32m to the left of the origin.

Question 40 (5 marks)



NOT TO SCALE

The diagram shows the graph of the curve $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(a) Show that the shaded region bounded by the curve, the x axis and the line

$x = k$, where $k > 0$, has area $\ln\left(\dfrac{e^k + e^{-k}}{2}\right)$. **2**

$$\int_0^k \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \left[\ln(e^x + e^{-x})\right]_0^k$$

$$= \ln(e^k + e^{-k}) - \ln(e^0 + e^0)$$

$$= \ln(e^k + e^{-k}) - \ln 2$$

$$= \ln\left(\frac{e^k + e^{-k}}{2}\right)$$

(b) Find, in simplest exact form, the value of k such that the shaded region has area of 1 square unit. **3**

$\ln\left(\frac{e^k+e^{-k}}{2}\right)=1$

$e=\frac{e^k+e^{-k}}{2}$

$2e=e^k+e^{-k}$

Multiply each term by e^k:

$2e(e^k)=(e^k)^2+(e^{-k})(e^k)$

$2e(e^k)=(e^k)^2+1$

$(e^k)^2-2e(e^k)+1=0$

let $m=e^k$

$m^2-2em+1=0.$

$m=\frac{2e\pm\sqrt{(-2e)^2-4(1)(1)}}{2(1)}$

$=\frac{2e\pm\sqrt{4e^2-4}}{2}$

$=\frac{2e\pm2\sqrt{e^2-1}}{2}$

$\therefore m=e\pm\sqrt{e^2-1}$

But $m=e^k$

$e^k=e\pm\sqrt{e^2-1}$

$\ln e^k=\ln\left(e\pm\sqrt{e^2-1}\right)$

$k=\ln\left(e\pm\sqrt{e^2-1}\right.$

End of Examination!!!

STUDENT NAME/NUMBER: solutions

Section I

10 Marks

Attempt Questions 1-10.

Allow about 15 minutes for this section.

Select either A, B, C or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the grid below.

This page must be handed in with your answer booklet.

	A	B	C	D
1				X
2	X			
3		X		
4		X		
5			X	
6				X
7			X	
8			X	
9	X			
10				X