BAULKHAM HILLS HIGH SCHOOL

2014
YEAR 11 YEARLY

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

Total marks – 70

This paper consists of TWO sections.

Section 1 – Multiple Choice
10 marks

Section 2 – Extended Response
60 marks
Attempt all questions
Start a new page for each question

Section 1 –Multiple Choice (10 marks)

Attempt all questions.

Answer the following on the booklet provided.

1 The number 0.09095 rounded to 2 significant figures is :

(A) 0.10 (B) 0.09 (C) 0.091 (D) 0.0910

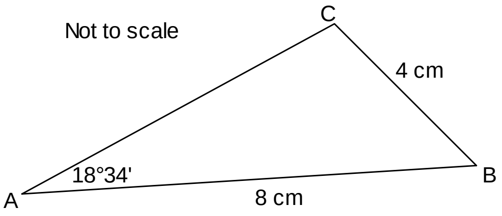
2 Simplify $5\sqrt{3}+\sqrt{20}-2\sqrt{12}+\sqrt{45}$

(A) $\sqrt{5}-\sqrt{3}$ (B) $\sqrt{5}+\sqrt{3}$ (C) $5\sqrt{5}+9$ (D) $5\sqrt{5}+\sqrt{3}$

3 If α and β are the roots of $15x^2+75x-3=0$, then $\alpha+\beta$ is :

(A) 75 (B) 5 (C) $-\frac{1}{5}$ (D) -5

4 A possible answer to the size of $\angle C$ in the triangle shown is :

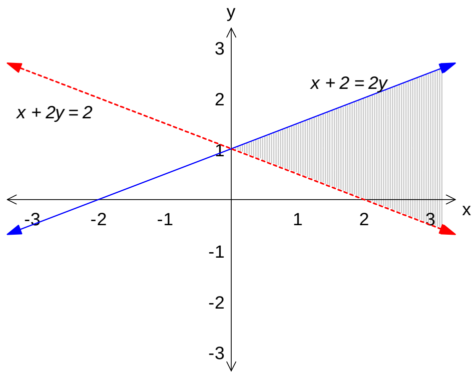


(A) $140°27'$ (B) $0^{\circ}10'$ (C) $37^{\circ}8'$ (D) none of these answers

5 What is the value of k if the expression $4x^2-6x+k$ is a perfect square ?

(A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) 4 (D) 9

6 The shaded region shown satisfies



(A) $x+2\geq 2y$ and $x+2y>2$ (B) $x+2\geq 2y$ and $x+2y<2$

(C) $x+2\leq 2y$ and $x+2y>2$ (D) $x+2\leq 2y$ and $x+2y<2$

7 If $2\cos x = \sqrt{3}$ in the domain $-180^{\circ} \leq x \leq 180^{\circ}$ then the values of x are :

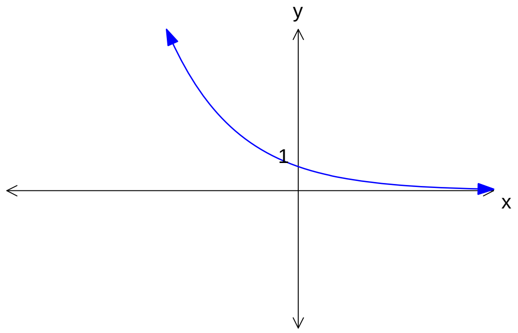
(A) $-60^{\circ}, 60^{\circ}$

(B) $-120^{\circ}, 120^{\circ}$

(C) $-30^{\circ}, 30^{\circ}$

(D) $30°, 150^{\circ}$

8 The graph illustrated could be :



(A) $y = 2^x$

(B) $y = (-2)^x$

(C) $y = \left(\frac{1}{2}\right)^x$

(D) $y = -2^{-x}$

9 The area of a rectangle with sides x and y is 47 cm^2. The perimeter can be represented by:

(A) $P = x + 47x^2$

(B) $P = x + \frac{47}{x}$

(C) $P = 2x + \frac{94}{x}$

(D) $y = 2x + \frac{47}{x}$

10 Find the domain over which the curve $y = x^3 + 3x^2 - 24x + 7$ is concave downwards.

(A) $x < -1$

(B) $-4 < x < 2$

(C) $x > -1$

(D) $x < -4, x > 2$

End of Section I

Section II – Extended Response
All necessary working should be shown in every question.

	Question 11 (15 marks) - Start on the appropriate page in your answer booklet	**Marks**
a)	Solve $|1-3x| > 13$	2
b)	Find a and b such that $\left(\sqrt{3}+4\right)^2 = a + b\sqrt{3}$	2
c)	For the function $y = \sqrt{9-x^2}$	
	(i) What is the domain and range ?	2
	(ii) Is the function odd, even or neither? (Justify your answer by showing working)	2
d)	The directrix of a parabola is the x axis and the focus is the point $(1, 4)$. Find the equation of the parabola.	2
e)	Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point where $x = 2$.	3
f)	Solve the pair of simultaneous equations $2x - y - 7 = 0$ $x + y + 1 = 0$	2
	End of Question 11	

Question 12 (15 marks) - Start on the appropriate page in your answer booklet		**Marks**
a)	Simplify $\frac{2x^3-16y^3}{x^2+2xy+4y^2}$	2
b)	Differentiate	
	(i) $\frac{2x+1}{x-1}$	2
	(ii) $x\sqrt{x}$	2
c)	The function $y = f(x)$ is given by $$f(x) = 3x(2x-1)^2$$	
	(i) Find the coordinates of the points where the curve $y = f(x)$ cuts the x axis.	2
	(ii) Find the coordinates of any stationary points on the curve $y = f(x)$ and determine their nature.	3
	(iii) Find any points of inflection.	1
	(iv) Sketch the curve $y = f(x)$ in the domain $-1 \leq x \leq 2$.	2
	(v) Hence find the maximum value for $y = f(x)$ in the given domain.	1
	End of Question 12	

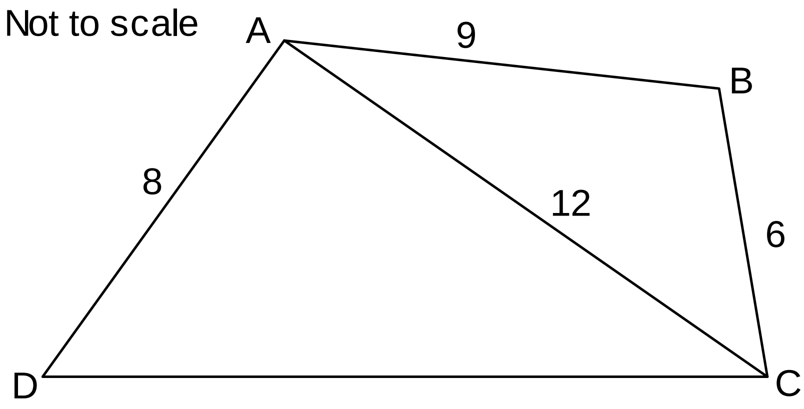
Question 13 (15 marks) - Start on the appropriate page in your answer booklet **Marks**

a) The quadratic equation, $P(x)$, is given by $P(x) = x^2 - 2(k-3)x + (k-1)$

(i) Find the value(s) of k for which $P(x) = 0$ has distinct real roots. 2

(ii) Explain what it means to the curve if $y = P(x)$ had been described as positive definite. 1

b) In the diagram $AB = 9\text{cm}$, $BC = 6\text{cm}$, $AD = 8\text{cm}$ and $AC = 12\text{cm}$.


Not to scale

(i) Find the size of $\angle ABC$, to the nearest degree. 2

(ii) Find the area of ΔABC, to one decimal place. 2

(iii) Given that $\angle ABC = \angle DAC$, prove that ΔABC is similar to ΔCAD. 3

(iv) Hence find the ratio of the area of ΔABC to the area of ΔDAC 2

c) Prove that 3

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$$

End of Question 13

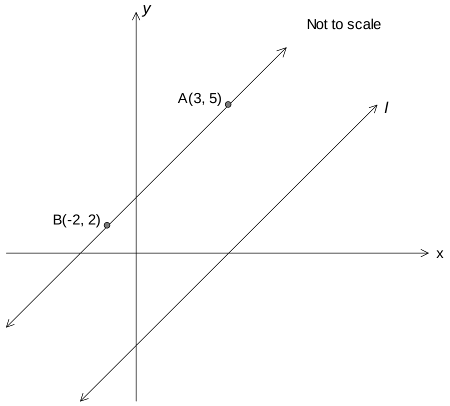
Question 14 (15 marks) - Start on the appropriate page in your answer booklet

a) Simplify

$$x^{-1}y^{2}(x^{\frac{1}{2}}-y^{-1})(x^{\frac{1}{2}}+y^{-1})$$ giving your answer with positive indices. 2

b) Given $3x^2+4x+5 \equiv A(x+1)^2+B(x+1)+C$, find the value of the constants A, B and C. 2

c) The line through $A(3,5)$ and $B(-2,2)$is parallel to the line l, $3x-5y-8=0$



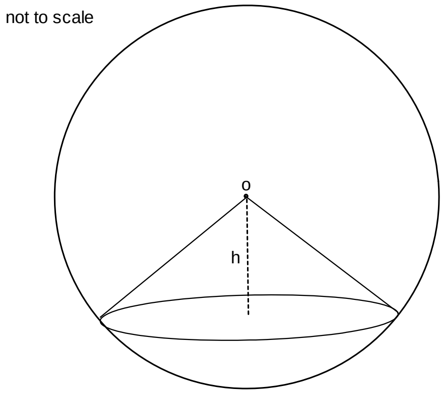
(i) Show that the equation of AB is $3x-5y+16=0$ 1

(ii) C is the point (1, -1) on line l. Find the perpendicular distance of C from the line joining A and B. 2

(iii) Find the area of the triangle formed by the points A, B and C. 2

(iv) Explain why the area of the triangle ABC is constant, regardless of the position of C on the line l. 1

d) A sphere contains a cone of height, h with its vertex at the centre of the sphere.
The radius of the sphere is 12cm.



(i) Show that the volume of the cone is $V=\frac{\pi}{3}(144h-h^3)$ 1

(ii) Given that the volume of the sphere is V_s,
show that the maximum volume of the cone is $\frac{\sqrt{3}}{18} \times V_s$ 4

End of Examination

Yr 11 Yearly 2014. – 2 unit

Section 1. (10 marks)

1. C	4. A	7. C	10. A
2. D	5. B	8. C	
3. D	6. A	9. C	

10 marks

Section II

Quest 11. (15 marks)

a) $|1-3x| > 13$

$1-3x > 13$ $\qquad$ $1-3x < -13$

$-3x > 12$ $\qquad$ $-3x < -14$

$x < -4$ or $x > \frac{14}{3}$.

① mark each correct section.

$\boxed{2}$

b. $(\sqrt{3}+4)^2 = a + b\sqrt{3}$

LHS $= 3 + 8\sqrt{3} + 16$

$= 19 + 8\sqrt{3}$

$\therefore a = 19 \quad b = 8$

① mark each value

$\boxed{2}$

c) $y = \sqrt{9-x^2}$

$9 - x^2 \geq 0$

i) D: $-3 \leq x \leq 3$

R: $0 \leq y \leq 3$

ii) $f(x) = \sqrt{9-x^2}$

$f(-x) = \sqrt{9-(-x)^2}$

$= \sqrt{9-x^2}$

since $f(x) = f(-x)$

$f(x)$ is an even function.

① correct domain

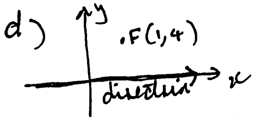
① correct range from above working

① stating even

① working

$\boxed{4}$

d)



$a = 2$

$V = (1,2)$

$\therefore (x-h)^2 = 4a(y-k)$

$(x-1)^2 = 8(y-2)$

① mark for finding 'a' or V.

① for correct equation

$\boxed{2}$

e) $y = 2x^2 - 5x + 1$

$\frac{dy}{dx} = 4x - 5$

at $x = 2$ $\quad m_1 = 8 - 5$ $\quad y = 2\times4 - 5\times2 + 1$

$= 3$ $\quad = -1$

$\therefore m_2 = -\frac{1}{m_1}$ $\quad (2,-1)$

$= -\frac{1}{3}$

$\therefore$ normal:

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{1}{3}(x-2)$

$3y + 3 = -x + 2$

$x + 3y + 1 = 0$

① finding y value

① finding m_2

① forming equation

$\boxed{3}$

f) $\begin{cases} 2x - y - 7 = 0 \\ x + y + 1 = 0 \end{cases}$

$2x - y = 7$ --- ①

$x + y = -1$ --- ②

① + ②

$3x = 6$

$x = 2$

sub into ①

$4 - y = 7$

$-y = 3$

$y = -3$

$\therefore$ Soln is $\underline{x = 2, y = -3}$.

② correct x and y.

① off for each mistake

$\boxed{2}$

Question 12 (15 marks)

a) $$\frac{2x^3-16y^3}{x^2+2xy+4y^2} = \frac{2(x^3-8y^3)}{x^2+2xy+4y^2}$$

$$= \frac{2(x-2y)\cancel{(x^2+2xy+4y^2)}}{\cancel{x^2+2xy+4y^2}}$$

$$= 2(x-2y)$$

① correct factorising

① answer

$\boxed{2}$

) i) $y = \dfrac{2x+1}{x-1}$

$$\frac{dy}{dx} = \frac{(x-1)(2)-(2x+1)\times 1}{(x-1)^2}$$

$$= \frac{2x-2\ -2x-1}{(x-1)^2}$$

$$= \frac{-3}{(x-1)^2}$$

① applying quotient rule

① answer.

ii) $y = x\sqrt{x} = x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} \text{ or } \frac{3\sqrt{x}}{2}.$$

① changing index power / applying product rule

① answer.

$\boxed{4}$

) $y = f(x) = 3x(2x-1)^2$

i) $f(x) = 0$ – x intercepts

$3x(2x-1)^2 = 0$

$x = \underline{0, \tfrac{1}{2}}.$ $\quad [(0,0)\ (\tfrac{1}{2},0)]$

① for each answer.

②

ii) Stat. pts.

$f(x) = 3x(2x-1)^2$

$f'(x) = 3(2x-1)^2 + 3x \times 4(2x-1)$

$= 3(2x-1)(2x-1+4x)$

$= 3(2x-1)(6x-1)$

$f'(x) = 0$ for stat pts

$\therefore (2x-1)(6x-1) = 0$

$x = \tfrac{1}{6}, \tfrac{1}{2}.$

① correct differentiation

① two correct points.

c) ii) cont.

$f''(x) = 6(6x-1) + 3\times 6(2x-1)$

$= 36x-6+36x-18$

$= \underline{72x-24}.$

$x = \frac{1}{6}$	$x = \frac{1}{2}$
$y = 3\times\frac{1}{6}(2\times\frac{1}{6}-1)^2$	$y = 3\times\frac{1}{2}(2\times\frac{1}{2}-1)^2$
$= 0.22$	$= 0$
$(\frac{1}{6}, 0.22)$	$(\frac{1}{2}, 0)$
test:	test:
$f''(\frac{1}{6}) = 72\times\frac{1}{6}-24$	$f''(\frac{1}{2}) = 72\times\frac{1}{2}-24$
$= -12 < 0$	$= 12$
max.	> 0 min
$(\frac{1}{6}, 0.22)$	$(\frac{1}{2}, 0)$ min

① for correctly testing both points.

iii) P.O.I.

$f''(x) = 72x - 24 = 0$

$x = \tfrac{1}{3}$ $\quad y = 3\times\tfrac{1}{3}(2\times\tfrac{1}{3}-1)^2$

$= 0.11$

$(\tfrac{1}{3}, 0.11)$

Test:

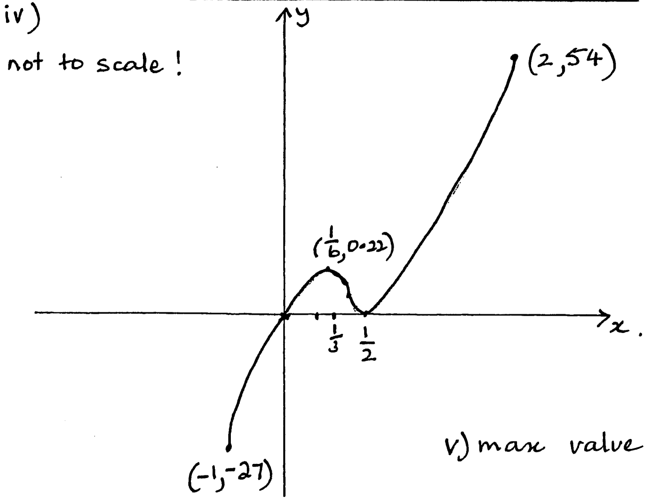
$f''(\tfrac{1}{6}) = -12 < 0 \qquad f''(\tfrac{1}{2}) = 12 > 0$

$\therefore$ a change in concavity.

① for correct POI and reason.

iv)

not to scale!



① for end points

① clear diagram

v) max value $y = \underline{54}$

① for correct value.

$\boxed{9}$

Question 13 (15 marks)

a) $P(x) = x^2 - 2(k-3)x + (k-1)$

i) $\Delta = b^2 - 4ac > 0$

$\therefore (-2(k-3))^2 - 4\times(k-1) > 0$

$4k^2 - 24k + 36 - 4k + 4 > 0$

$4k^2 - 28k + 40 > 0$

$k^2 - 7k + 10 > 0$

$(k-5)(k-2) > 0$

$\therefore \underline{k < 2 \text{ or } k > 5}$

① forming quadratic.

① correct solution.

ii) positive defn $\Delta < 0$

The curve $y = P(x)$ would be concave up and completely above the x axis

① logical explanation.

$\boxed{3}$

b) i) $A\hat{B}C$

$$\cos B = \frac{a^2 + c^2 - b^2}{2\times a\times c}$$

$$= \frac{9^2 + 6^2 - 12^2}{2\times 9\times 6}$$

$$= \frac{-27}{108}$$

B in 2nd quad.

$B = 180 - 75.52^\circ$

$= \underline{104^\circ}$ (nearest degree)

① applying cosine rule correctly.

① correct angle

ii) $A_{\triangle ABC} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 9\times 6\times \sin 104^\circ$

$= 26.197\ldots$

$= 26.2\text{ cm}^2.$

① applying area formula.

① correct answer.

13. b) iii)

In $\triangle ABC$ and $\triangle CAD$

$$\left.\begin{array}{l} \dfrac{BC}{AD} = \dfrac{6}{8} = \dfrac{3}{4} \\[2ex] \dfrac{BA}{AC} = \dfrac{9}{12} = \dfrac{3}{4} \end{array}\right\} \text{same ratio}$$

$\angle DAC = \angle ABC$ (given)

$\therefore \triangle ABC \,|||\, \triangle CAD$ (2 pairs of sides in ratio included $\angle$ equal)

① establishing ratios.

① angle.

① reason

iv) $A_{\triangle ABC} : A_{\triangle DAC}$

since $\dfrac{S_1}{S_2} = \dfrac{3}{4}$ (ratio found in proof)

then $\dfrac{A_1}{A_2} = \dfrac{S_1^2}{S_2^2}$

$= \dfrac{9}{16}$

or $A_{\triangle ABC} : A_{\triangle DAC} = \cancel{\frac{1}{2}}\times 9\times 6\times \cancel{\sin 104} : \cancel{\frac{1}{2}}\times 8\times 12\times \cancel{\sin 104}$

$= 9\times 6 : 8\times 12$

$= \underline{9 : 16}$

or $A_{\triangle ABC} = 26.2$ (from above)

$A_{\triangle DAC} = \frac{1}{2}\times 8\times 12\times \sin 104$

$= 46.57.$

$\therefore 26.2 : 46.57 = 5625 : 100000$

$= 9 : 16.$

② using one of these methods to establish correct ratio.

$\boxed{9}$

13 c)

AIM: prove

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\sin\theta+\cos\theta .$$

$$LHS=\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}$$

$$=\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}+\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}$$

$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}+\frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)\cancel{(\cos\theta-\sin\theta)}}{\cancel{\cos\theta-\sin\theta}}$$

$$=\cos\theta+\sin\theta$$

$$=\underline{\underline{RHS}}.$$

① off each error.

① linking LHS to RHS. (concluding proof)

max $\boxed{3}$ marks.

Question 14 (15 marks)

a) $$x^{-1}y^2(x^{\frac{1}{2}}-y^{-1})(x^{\frac{1}{2}}+y^{-1})=\frac{y^2}{x}(x-y^{-2})$$

$$=\frac{y^2}{x}\left(x-\frac{1}{y^2}\right)$$

$$=\frac{\cancel{y^2}}{x}\left(\frac{xy^2-1}{\cancel{y^2}}\right)$$

$$=\frac{xy^2-1}{x}$$

① correctly expanding brackets

① correct final line.

$\boxed{2}$

b) $$3x^2+4x+5\equiv A(x+1)^2+B(x+1)+C$$

$$RHS=Ax^2+2Ax+A+Bx+B+C$$

$$=Ax^2+x(2A+B)+A+B+C$$

$\therefore 3=A$

$4=2A+B$

$4=6+B$

$B=-2$

$5=A+B+C$

$5=3-2+C$

$4=C$

$\therefore A=3,\ B=-2,\ C=4$.

② for 3 correct values.

① for setting up equations and finding 1 correct value

$\boxed{2}$

c) AB.

i) $m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{5-2}{3+2}$

$=\frac{3}{5}$

(−2, 2)

$y-y_1=m(x-x_1)$

$y-2=\frac{3}{5}(x+2)$

$5y-10=3x+6$

$3x-5y+16=0$.

① for establishing the equation!

ii) $$\text{perp } d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{3\times1-5\times-1+16}{\sqrt{9+25}}\right|$$

$$=\left|\frac{3+5+16}{\sqrt{34}}\right|$$

$$=\frac{24}{\sqrt{34}}\text{ units}$$

① correctly substitute into formula.

① correct answer.

c iii)

$$\text{dist}_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34} \text{ units}$$

① dist AB

$$A_{\triangle} = \tfrac{1}{2}bh$$

$$= \tfrac{1}{2} \times \cancel{\sqrt{34}} \times \frac{24}{\cancel{\sqrt{34}}}$$

$$= 12\,u^2$$

① correct Area.

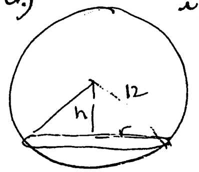
$\boxed{2}$

iv) The line l, is a fixed distance from AB as it is parallel to AB. Therefore the perpendicular height is fixed and the base is fixed as A to B ∴ Area is constant.

① the idea of parallel = fixed height

$\boxed{b}$

d.)



i)

$$h^2 + r^2 = 12^2$$

$$r^2 = 144 - h^2$$

$$V = \tfrac{1}{3}\pi r^2 h$$

$$= \tfrac{\pi}{3}(144 - h^2)h$$

$$= \tfrac{\pi}{3}(144h - h^3)$$

① for finding r^2

ii)

$$V_s = \tfrac{4}{3}\pi r^3$$

$$= \tfrac{4}{3}\pi \times 12^3$$

$$= 2304\pi.$$

$$V_{cone} = \tfrac{\pi}{3}(144h - h^3)$$

$$\frac{dV_c}{dh} = \tfrac{\pi}{3}(144 - 3h^2) \qquad \frac{d^2V}{dh^2} = \tfrac{\pi}{3}(-6h)$$

now $\frac{dV}{dh} = 0$

$$\tfrac{\pi}{3}(144 - 3h^2) = 0$$

$$144 = 3h^2$$

① correct derivative of V_c.

$$h^2 = 48$$

$$h = \pm\sqrt{48}$$

now $h > 0$

$$\therefore h = \sqrt{48}$$

$$= 4\sqrt{3} \text{ cm.}$$

① correct h.

Test for max.

$$\frac{d^2V}{dh^2} = \tfrac{\pi}{3}(-6h)$$

$$= \tfrac{\pi}{3}(-6 \times 4\sqrt{3}) < 0$$

∴ max.

① testing max/min.

$$\therefore V_c = \tfrac{\pi}{3}(144h - h^3)$$

$$= \tfrac{\pi}{3}(144 \times 4\sqrt{3} - (4\sqrt{3})^3)$$

$$= \tfrac{\pi}{3}(576\sqrt{3} - 192\sqrt{3})$$

$$= \tfrac{\pi}{3} \times 384\sqrt{3}$$

$$= 128\sqrt{3}\pi$$

$$\therefore V_s = 2304\pi$$

$$\frac{\sqrt{3}}{18} \times V_s = \frac{2304\pi \times \sqrt{3}}{18}$$

$$= 128\sqrt{3}\pi = V_c.$$

① finding concluding Volume ratio.

or.

$$\frac{V_c}{V_s} = \frac{128\sqrt{3}\cancel{\pi}}{2304\cancel{\pi}}.$$

$$= \frac{128\sqrt{3}}{2304}$$

$$= \frac{\sqrt{3}}{18.}$$ as required.

$\boxed{5}$